The fair value of a mortgage

Anatoliy Babič Mihail Bazhba Felix Beckebanze
Marko Dimovski Aldina Correia Eliana Costa e Silva
Marko Dimovski Robert Fitzner Stella Kapodistria*
David Koops Willem Moerkens
Irving van Heuven van Staereling Xiaoming op de Hoek
Bart Sevenster Jok Tang

Abstract

We consider the problem proposed by the Bank at the SWI 2017 meeting. In particular, the following directions of investigation were proposed:

Question 1: How should the bank calculate the fair value for it's current portfolio?

Question 2: What are the main drivers for prepayments?

Question 3: What is the biggest concern for the bank with the present low/negative rates regarding mortgages? How should the bank deal with this?

Question 4: What can the bank do to mitigate the risk of prepayments?

Question 5: What is the fair value of the banks portfolio (Dataset 2B)?

In this report, we deal with the mathematically oriented questions and we are interested in the modelling of the prepayments and their prediction.
KEYWORDS: mortgage, prepayments.

1 Introduction

Mortgages are an important tool in making housing available for people who do not have sufficient savings but still want to buy a house. In the Dutch economy, mortgages play an important role in consumer expenditure. The total household debt in the Netherlands in 2011 was 117% of GDP according to CBS, see (2). Household debt (including debt of non-profit institutions serving households) are the loans on the liabilities side of the balance sheet, excluding mutual household debts. While the non-financial corporations accounted for 107% of the GDP, (2). The bulk of household debts consists of residential mortgages. The mortgage debt has grown substantially between 1995 and mid-2012. After a period of decline between 2012 and 1014, household debts started rising as of September 2014, in particular the level of residential

*s.kapodistria@tue.nl

mortgage debt. The latter increased from 649 billion euros at the end of September 2014 to 669 billion euros at the end of June 2017. In the same period, non-mortgage debt rose from 88.5 to 91 billion euros. Despite increasing debt levels, household debt as a percentage of GDP declined in Q2 2017, because the increase in GDP was stronger than the increase in debt levels. At the end of June 2017, the household debt ratio had declined to 106.1 percent from 106.7 percent in March; the ratio has been falling since Q4 2012. In Q2 2017, the non-financial sector debt ratio declined from 114.1 to 112.7 percent. Total private sector debt amounted to 218.8 percent as a result, the lowest level since 2008.

Mortgages are for the bank an investment, as the bank loans the money in the form of a mortgage to the customer with a given interest rate. Although mortgages are a relatively secure investment opportunity due to their structured payment scheme and the interest rate which is fixed for a long time, mortgages do possess some risks for the issuer of the mortgage, typically the bank, which need to be taken into account when assessing the value of a mortgage. The most important risks related to mortgages are the risk of defaulting, which is the risk of customers not paying back their mortgage, and the interest rate risk, which is the risk caused by the uncertainty of future interest rates. These risks have been extensively researched both in practice and in the academic literature. A relatively less known risk for the issuer is that of the prepayment risk, which is the risk created by customers paying back (partially or in full) their mortgages earlier than the date stated in their contract. Prepayments can be done after the customers sell the house or refinance the mortgages, after which the mortgage ceases to exist. Refinancing can be lucrative if interest rates are low. It is also possible that customers pay back only a small percentage of the mortgage debt in order to reduce the interest that they have to pay, while staying under the extra instalments threshold so as to avoid penalties associated with prepayments.

A bank needs to meet its obligations: similarly to individuals, a bank needs to meet all its anticipated expenses, which in this case are the funding of loans, making payments on debt, etc. These payments need to be done by the bank using liquid assets, e.g., money. Ideally, a bank should maintain a level of liquidity that also allows it to meet any unexpected expenses without having to liquidate other assets. The bigger the cushion of liquid assets relative to anticipated liabilities, the greater the bank's liquidity. To this end, the bank makes a financial plan based on payments it will receive and the payments it needs to make. Thus, if many people simultaneously prepay their mortgage, the bank is thrown off this financial balance. Furthermore, because of the high costs of acquiring a mortgage, mortgages typically need to remain on the books for several years in order to be profitable.

A very recent trend noticed in the financial markets is that of the reduction of the average tenure of a residential mortgage (i.e., duration of a mortgage at one issuer), due mainly to consumers' increased willingness to switch lenders for a better deal. This is a worrisome trend for lenders, as they are exposed due to lack of liquidity and

due to the unprofitability of mortgages for the banks when they are prepaid at very short tenure.

In general, the following four drivers are shown to have a significant impact on prepayment behaviour, see (5):

- Age of the fixed rate loan – Typically, prepayment occurs after the end of the first fixed rate period.

- House price inflation – When house price inflation is high, the number of home moves increases. Increased activity in the housing market results in increased prepayment.

- Interest differential – This measures the tangible saving that a borrower could make by switching to another fixed rate or variable rate mortgage (typically by negotiating the mortgage anew with another bank - refinancing).

- Prepayment charges – These charges create a cost to prepayment that acts as a disincentive to prepay. We observed that charges over a certain level appeared to discourage prepayment significantly.

The goal of this report is to propose some models for the calculation of the fair value of a mortgage i.e. the value of a mortgage when taking prepayment risk into consideration.

Section 2 introduces the notation used in this analysis, Section 3 describes some common mortgage types and their characteristics. In Section 5, the risks for a bank are described. Section 6 introduces the way prepayments can be modelled and Section 7 introduces some models to incorporate prepayments in the valuation of a mortgage portfolio.

2 Prerequisites

Throughout this study, we consider mortgages with a contractual duration of M months, an initial coupon c (interest rate, expressed in % per month), and an initial principal P_0. For simplicity, we assume that the maximum duration for a mortgage is 30 years, i.e., $M \leq 360$. Furthermore, without loss of generality, we assume that the interest rate, c, is fixed throughout the entire contractual period, except if stated otherwise. We express the contractual payment (cashflow) by the borrower to the bank in month t by x_t. The amount of the monthly payment will be calculated in accordance to the type of mortgage.

3 Types of mortgages

Bullet mortgage

Bullet mortgages do not require the pay-off of the initial principal, P_0, throughout the contractual period. The monthly payment, x_t, thus only consists of the interest payment, i.e. $x_t = cP_0$ for all $t = 1, 2, \ldots, M-1$. The only exception is the last payment, $t = M$, in which the initial principal needs to be repaid, hence $x_M = P_0(1+c)$, see figure 1.

Linear mortgage

In a linear mortgage, the borrower repays the initial mortgage loan by a fixed amount every month. On top of this the borrower pays interest, but the interest payments will reduce over time since the borrower is gradually redeeming the initial loan. Since the mortgage amount will actually decrease, so will the interest payments. Say that the initial principle is P_0. Then, each month a payment of P_0/M is performed, plus the interest. So the monthly payment, x_t, is thus the sum of P_0/M plus the interest, cP_t, with $P_t = \frac{M+1-t}{M} P_0$. The monthly interest decreases every month, since the principle, P_t, decreases over time.

Straight line or level-pay or annuity mortgage

The characteristic of a straight line mortgage is that the monthly payment by the borrower is constant, x_* (assuming that there are no fluctuations in the mortgage interest rates). This means that initially the borrower pays a lot of interest, while the pay-off of the initial principle, P_0, is relatively small. This reverses towards the end of the mortgage term, when a smaller fraction of the monthly payment consists of interest payment. The constant monthly payment, x_*, can be calculated as follows

$$P_0 = \left(\sum_{t=1}^{M} \frac{x_*}{(1+c)^t} \right)^{-1} \Rightarrow x_* = \frac{cP_0(1+c)^M}{(1+c)^M - 1}. \tag{1}$$

The situation in the Netherlands

The Dutch housing market is shaped by four dominant forces, which stem from former political choices, see (3): (i) income tax deductibility of mortgage interest; (ii) a rental market in which not-for-profit social housing institutions have a combined market share of 84 percent; (iii) a scheme involving rent control and strong tenant protection; and (iv) a restrictive regulatory ('zoning') regime for the development and construction of new homes. The Dutch housing stock consists of 7.1 million houses, 56 percent of which are in the owner-occupied segment. This rate is below average in the eurozone. The low share of the owner-occupied segment in itself acts as

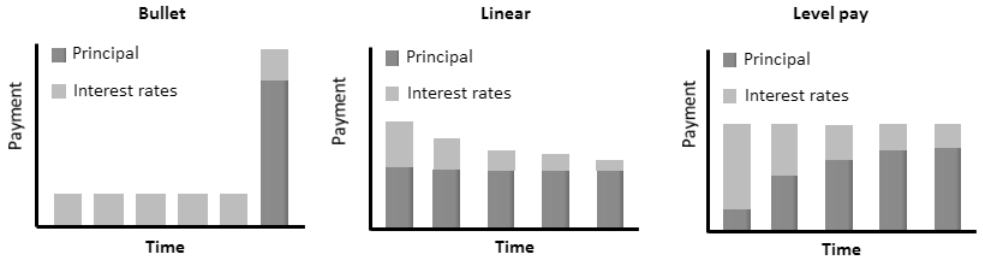


Figure 1: Cashflow (monthly payments) depiction for the various types of mortgages: Bullet, Linear and Level Pay mortgage

risk filter, since access to ownership is restricted to households with a good risk profile.

The Netherlands scores high in terms of mortgage debt. In fact, with a mortgage debt stock equalling 108% of the gross domestic product in 2012, see Figure 2. On the basis of this high debt burden, risks to the Dutch mortgage market are perceived to be elevated.

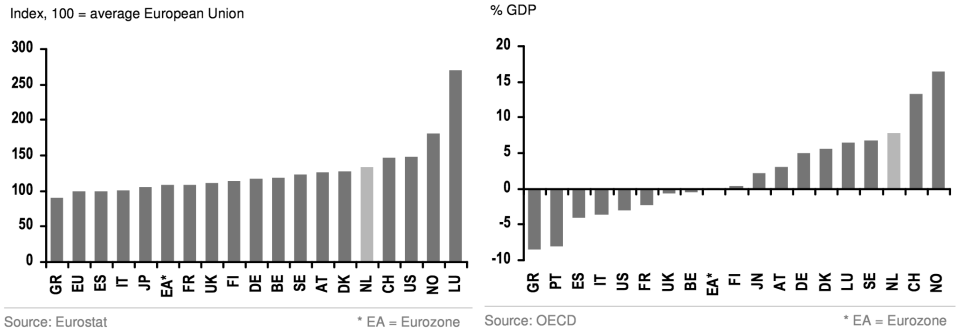


Figure 2: GDP per capita (2010), on the left, and current account balances (2011), on the right

In (1), the following conclusions were drawn for the Dutch mortgage market:

- The strong asset base of Dutch households and the full tax deductibility of interest payments are the primary reasons for the relatively high mortgage debt levels in the Netherlands.
- From 2013 onwards, it is expected that tax deductibility will only apply to amortising mortgage loans. Existing mortgage borrowers will be excluded from this change and continue to benefit from the existing tax regime. First-time

buyers will be hit by this change, which translates into higher net mortgage servicing costs.

- The Dutch housing market is unlikely to recover in the short run. Cyclical headwinds to the economy, a very low level of consumer confidence and structural changes to the housing and mortgage market are making people reluctant to buy a house. House prices are likely to decline further in the short run.

- Foreclosures rates are very low, especially in international comparisons. Although the modest recession is likely to result in an increase in the foreclosure rate, the resilient structure of the economy and the mortgage market will prevent a sharp increase in mortgage defaults.

Risks for the Dutch mortgage market

Till 2012, the main risks for the issuer of a mortgage loan were late payments and ultimately foreclosure. Late payments are generally managed well. Virtually all mortgage payments are automatically debited from current accounts. Payment failures are quickly discovered and notices are sent out usually within days. Statistics from various rating agencies show that mortgage arrears are very low in the Netherlands. By international comparison, both late payment and foreclosure rates are among the lowest in Europe.

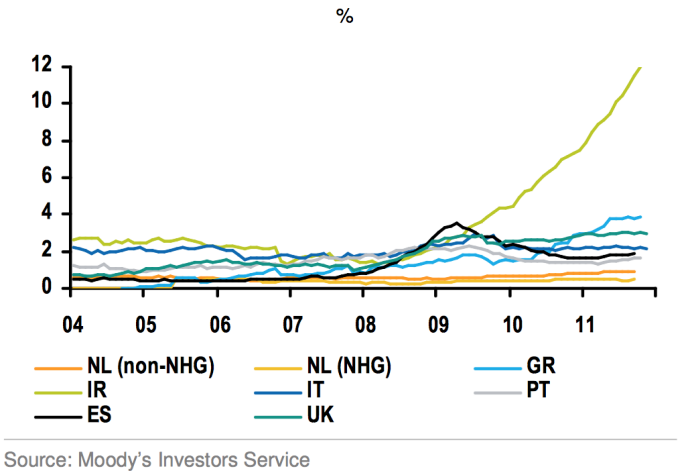


Figure 3: RMBS Prime 60+ days delinquency ratet

A very recent factor of risk is related to the housing price, (3). For about thirty percent of Dutch mortgages, the size of the mortgage exceeds the value of the un-

derlying property (4)[1]. These households suffer from negative home equity, their mortgages are 'underwater'. (4) shows that the underwater problem affects mainly younger households (20-40 years).

Key figures for the Dutch mortgage market

According to (3), the tax deductibility of mortgage interest has greatly influenced the Dutch mortgage market. It has encouraged 'interest only' mortgages, leading to high portfolio LTVs, and caused a large difference between LTIs based on gross and net income. Yet defaults and losses have remained very low, even in the recent crisis. The mortgage portfolio of lenders in the Dutch market consists of 3.5 million households: 83 percent of the 4.3 million Dutch homeowners carry a mortgage debt, with their property as collateral. In 2013, the total mortgage debt amounted to EUR 637 billion. The value of the housing stock amounts to EUR 1.07 billion.

4 Mortgage value

The objective of this study is to model and predict the *net present value* of a mortgage portfolio. The net present value of a mortgage portfolio is the sum of all future cash flows, $x_{i,t}$, of mortgage i at time t, discounted appropriately. In order to be able to generate income, the interest rate at which the bank borrows money, say r_t (expressed in % per month and called yield), must be smaller than the coupon rate, c_t, of the mortgages.

Let c_t denote the cashflow at time t, then its net present value at time 0 is given by $\frac{c_t}{(1+r_{\text{eff}})^t}$, assuming that r_{eff} is the effective interest rate (corresponding here to a monthly period). If the interest rate changes over time, then the net present value can be calculated by discounting every month and using the appropriate interest rate as they appear in the yield curve, see Figure 4.

We assume that the future cashflows from a borrower, c_t, are always larger or equal to the contractual payments, x_t. The exact amount of the additional payment, $d_t = c_t - x_t$, which is voluntary, depends on various parameters, which are discussed in more detail in §6. The net present value for a mortgage at time $t = 0$, (PV), with future cash flows c_t for $t = 1, .., M$ is given by

$$PV = \sum_{t=1}^{M} \frac{c_t}{(1 + r_{\text{eff}})^t} \tag{2}$$

[1]This figure is based on an approximated correction for mortgage-related savings (cf. Chapter 3). Without this correction, and including non-bank loans, the percentage of underwater loans amounts to 41 percent (2).

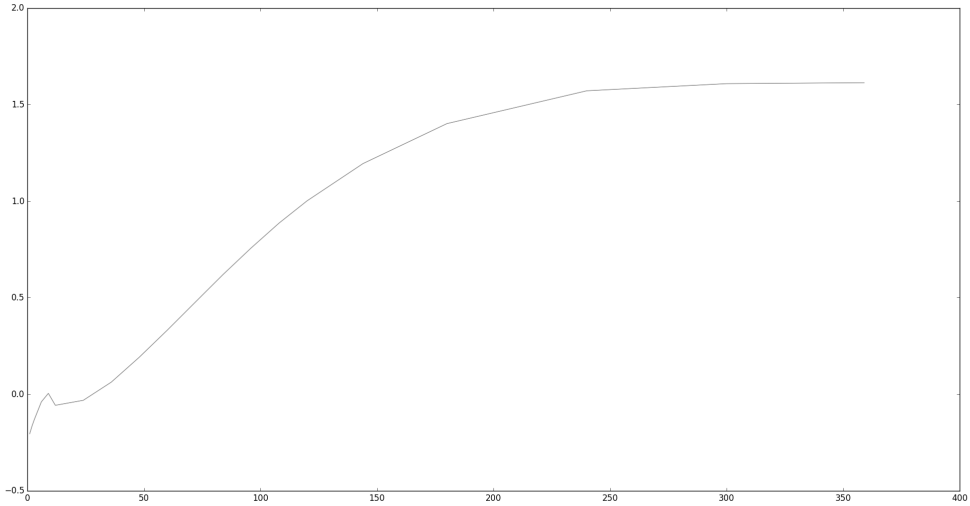


Figure 4: *Interpolated yield curve, 01-01-2016. Yield versus number of months (0 to 360).*

for a constant yield r_{eff}. If $r = r(t)$ varies over time, then

$$PV = \sum_{t=1}^{M} c_t \prod_{t'=1}^{t} \frac{1}{1 + r(t')}.$$

5 Risks of a mortgage portfolio

The net present value of a mortgage portfolio is the value of each discounted future cash flow of the mortgages. However, there are several risks that make valuation of the portfolio complicated; some of these risks are listed below.

- **Default risk** Default risk is the risk of customers not paying back (part of) their mortgages. This can be the result of people losing their jobs or having other problems with income which means they do not have enough money left to pay off their mortgages. Another reason is people dying before the end of the mortgage term.
- **Interest rate risk** Interest rate risk is the risk that banks have because of changing market interest rates and with it changing income and expenses. The interest that the banks receive on the loans and mortgages they have and the interest they have to pay on the savings accounts of their customers depend heavily on the market interest rate. A change in the market interest rate therefore influences the ratio of income and expenses.

- **Prepayment risk** Related to interest rate risk is the prepayment risk. Prepayment risk is the risk that customers pay back their loans or mortgages earlier than their contract says they have to, which means that the bank misses out on interest they would have received if no prepayment had been done. The prepayment risk depends on the interest rate since lower interest rates can make it more profitable for customers to pay back part of their mortgages, since the money they put on a bank account does not yield enough interest any more.

- **Reputational risk** Reputational risk is the risk that a bank has based on its reputation as a reliable business partner. If the bank has a bad reputation, customers may not want to use any of the financial products that the bank has to offer and instead go to one of its competitors.

- **Operational risk** Operational risk is the risk the bank has in its operations. According to Solvency 2 operations risk is "the risk of a change in value caused by the fact that actual losses, incurred for inadequate or failed internal processes, people and systems, or from external events (including legal risk), differ from the expected losses".

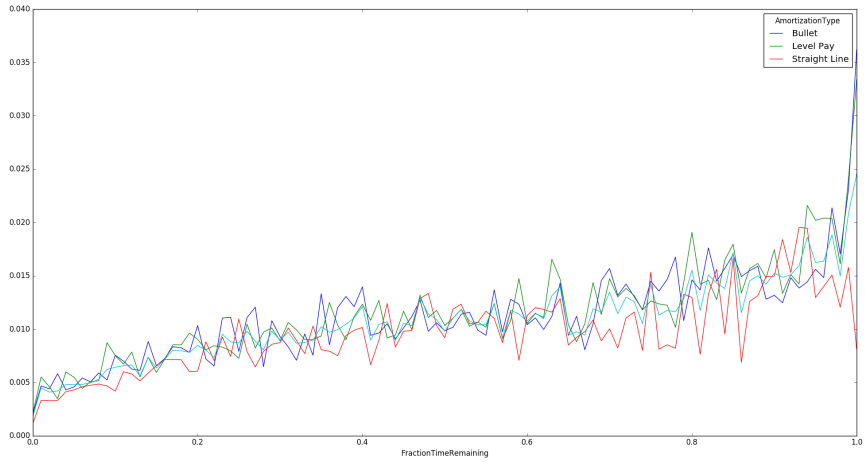


Figure 5: *Average fraction of outstanding principal that is prepaid, versus the fraction of time that is remaining until maturity, for each different type of mortgage (bullet, level pay, straight line). The light blue plot shows the average over all of these types.*

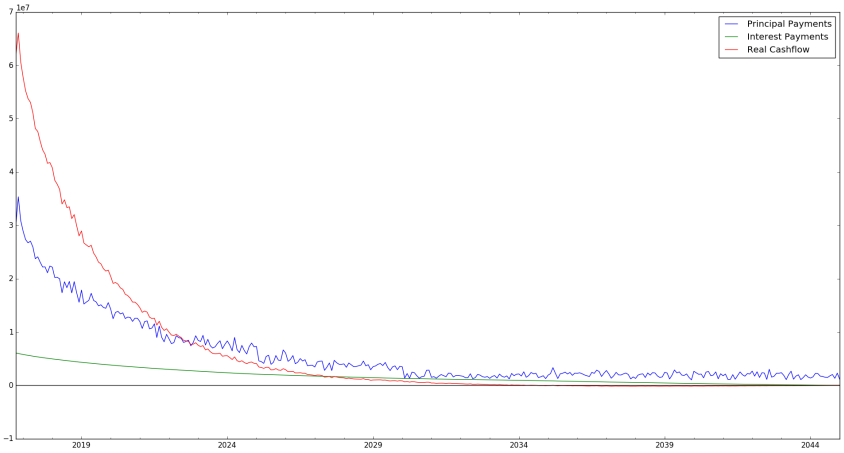


Figure 6: *The red curve is the (expected real (non-discounted) cash flow, based on prepayment model. The blue curve are the remaining principals according to the contractual payments. Green represents the interest payments.*

6 Prepayment modelling

In this section we zoom in on the prepayment risk, which is a significant risk in a mortgage portfolio. In order to manage this risk, it is important to have insight into the prepayment behaviour of clients. Therefore, we used the provided data[2] to find the relation between the duration (defined as the time in months since the start of the amortisation), and the average prepayment rate (defined as the prepayment made as a fraction of the outstanding principal, averaged over all contracts). This relation is represented in Figure 5. We see that in the beginning of the period, people typically prepay a smaller amount of their outstanding principal. We can fit a curve through this (for each type of mortgage, or just one curve if the differences are negligible). This curve can be used to generate the future expected cashflows, which consists of the contractual cashflows, in addition to the expected prepayments. The result of this is depicted in Figure 6. As a final step, we have to discount these cashflows by making use of the yield curve in order to obtain the present net value. The yield curve that we are provided with for the present time, does not contain a value for each month, but there are gaps. Therefore we interpolate the yield curve, after which we can use

[2]Due to confidentiality reasons and in order to guarantee the anonymity of the Bank that provided us with the data set, we cannot describe the data, but can only present some results obtained from the data set. We would also like to note that these results might be specific to the dataset we analysed and might not be replicable for other data sets, as the data set provided to us was synthetic.

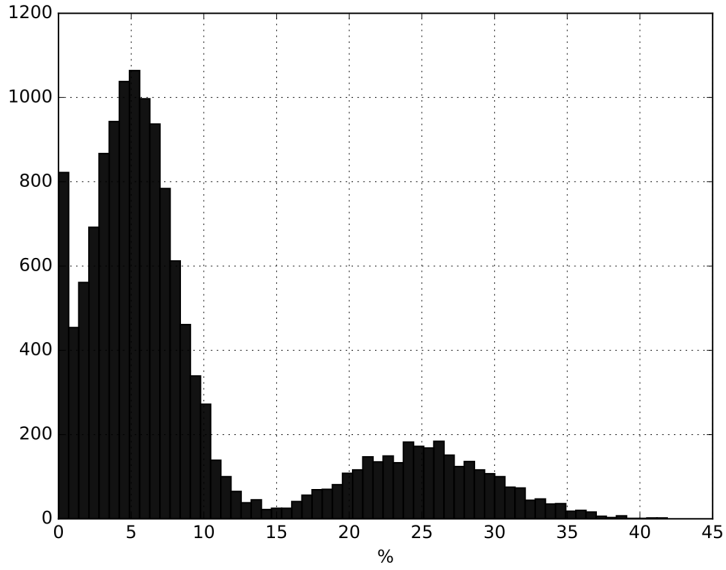


Figure 7: *Each bar represents the count of the number of people who prepay the corresponding percentage of their outstanding principal.*

it for discounting the future cash flows. The interpolated yield curve is depicted in Figure 4.

7 Improvements of the model

We could make improvements on the model by not only distinguishing between the different type of mortgages, but by also considering different types of prepayments (i.e., curtailment, relocation, refinancing). Within curtailment, we can make a further distinction, which is suggested by Figure 7. The low number of people who make prepayments at around 15% of the outstanding principal can be explained by the fact that there is a penalty inflicted on people who pay off more than 15% of their outstanding principal. Therefore, people who would like to make a curtailment, either avoid it by paying less than 15%, and if they really want to make a larger curtailment they make it worthwhile by substantially overshooting the 15% barrier.

8 Relocation distribution – Fitting Approach

This study focuses on the relocation cases for the mortgage portfolio of the bank, with interest time (maturity) equal to 360 months. By visual inspection, we consider the Gamma distribution as the best candidate to fit the data of the variable *ElapsedTime*,

see Figure 8. This variable represents the number of months elapsed at the time a client decides to relocate this mortgage. Other distributions were tested, such as the log-normal and beta, but the gamma distribution had the best fit.

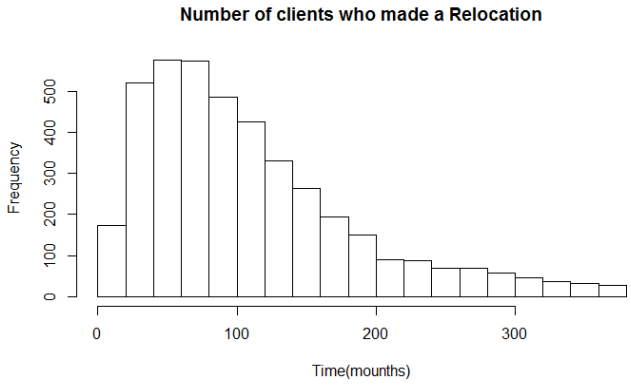


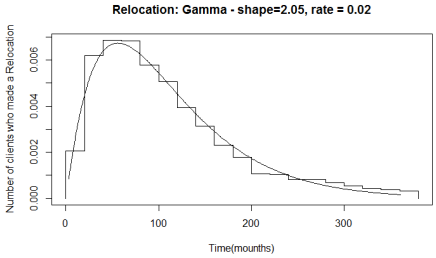
Figure 8: Histogram of the variable "Number of Months" accounting for the months that have elapsed since the beginning of the mortgage

Let $\mathbf{Y}$ be a random variable with the Gamma distribution with parameters ν and ν/μ, symbolically $\mathbf{Y} \sim Ga(\nu, \nu/\mu)$, then the density function of this random variable is written as

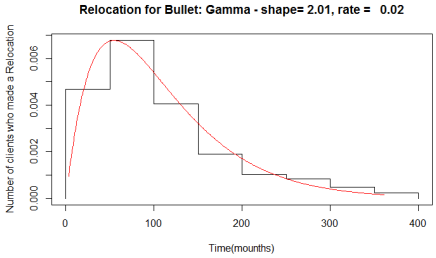
$$\begin{aligned} f(y|\nu,\mu) &= \frac{1}{\Gamma(\nu)}\left(\frac{\nu}{\mu}\right)^{\nu} y^{\nu-1}\exp\left(-\frac{\nu}{\mu}y\right) \\ &= \exp\left\{\nu(\theta y + \log(-\theta)) + (\nu-1)\log(y) - \log(\Gamma(\nu)) + \nu\log(\nu)\right\}, \end{aligned}$$

with $y > 0$, $\theta = -1/\mu$ and Γ the gamma function.

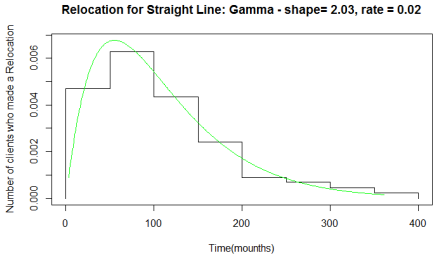
We can divide the data in the mortgage portfolio in 3 groups distinguishing for the three types of mortgages: Bullet (1401 clients), Level Pay (1404 clients) and Straight Line (1400 clients). In the first fit, we consider all three types of mortgages, obtaining the parameter estimates $\gamma = 2.0492072693$ and $\gamma/\mu = 0.0186694832$, see Figure 9a. If we consider just one type of mortgage, e.g. Bullet, we can repeat the fitting and obtain very similar results. More concretely, we obtain the following parameter estimates $\gamma = 2.0104627616$ and $\gamma/\mu = 0.0185100704$, see Figure 9b. If we consider the Straight Line, we obtain the following parameter estimates $\gamma = 2.0288771200$ and $\gamma/\mu = 0.0186201469$, see Figure 9c, and for Straight Line, we have the following estimates $\gamma = 2.1129085041$ and $\gamma/\mu = 0.0189148191$, see Figure 9d.



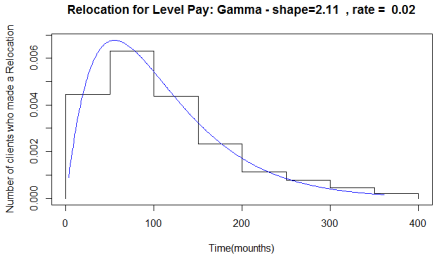
(a) $\gamma = 2.0492072693$ and rate $\gamma/\mu = 0.0186694832$



(b) Gamma $\gamma = 2.0104627616$ and rate $\gamma/\mu = 0.0185100704$ shaped histogram and fit curve - Bullet Amortisation Type



(c) Gamma $\gamma = 2.0288771200$ and rate $\gamma/\mu = 0.0186201469$ shaped histogram and fit curve - Straight Line Amortisation Type



(d) Gamma $\gamma = 2.1129085041$ and rate $\gamma/\mu = 0.0189148191$ shaped histogram and fit curve - Level Pay Amortisation Type

Figure 9: Fitting the data to the Gamma distribution with parameters ν and ν/μ

Using the well known Kolmogorov-Smirnov test (KS), we tested the similarities between the various estimates for the parameters comparing the case that all the data is aggregated versus the cases that we distinguish for each type of mortgage. In all cases, a significance level $>> 5\%$ was obtained, therefore we can consider that there exist no significant differences between the 4 distributions. Thus, the distribution of the months since the beginning of the mortgage to relocation does not depend on the type of amortisation and the same probability distribution can be used for all the amortisation types.

9 Relocation distribution – Survival Approach

In this section, we investigate the distribution of the relocation lifetime using notions from survival analysis. More concretely, we use the Kaplan-Meier (KM) estimator, which is a non-parametric statistic for the estimation of the survival function from

the relocation amortisation lifetime data. In the present analysis, considering as zero the instant in time at which the mortgage is contracted, the KM estimator gives, at each month after the begging of the mortgage, an estimation of the probability that a client has not relocated. The obtained KM estimator is illustrated in Figure 10.

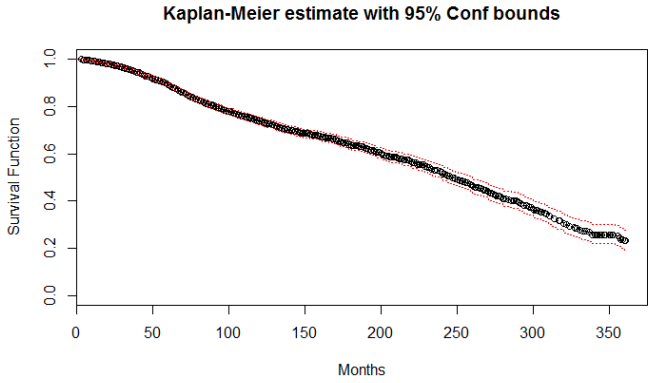


Figure 10: Kaplan - Meier estimator

References

[1] Ruben van Leeuwen and Philip Bokeloh (2012). *Mortgage Market in the Netherlands*, ABN AMRO Bank N.V., The Netherlands.

[2] CBSlink - Financial balance sheets and transactions by sectors; National Accounts http://CBSlink.cbs.nl/ (link).

[3] Dutch Banking Association (2014). *The Dutch Mortgage Market*, Nederlandse Vereniging van Banken, The Netherlands.

[4] Dutch Central Bank (2014). *Overview of Financial Stability*, Amsterdam: DNB.

[5] Simon Perry, Stuart Robinson, and John Rowland (2001). A study of mortgage prepayment risk. *Housing Finance International*, 16(2), 36–51.