

# Strategic bidding in a primary reserve auction

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## 1 Introduction

Electricity grids are subject to a constant change of demand. If a power line is overloaded, the demand is rerouted to another line, which is then also likely to overload due to the sudden spike in voltage. Due to this cascading effect a grid-wide blackout is not at all improbable; one occurred in Italy in 2003. The costs of such a blackout are immense in today's modern society. Transport and telecommunication systems have such a high power demand that a backup power generator system would come at a very high cost. To solve this, Germany requires the electricity grid operators to have Primary Reserve Capacity on standby. Primary Reserve Capacity is provided by power plants that are able to go online and start generating power at 10 seconds notice. Such a feat is technically complicated, and providing plants require government certification. In Germany the PRC is obtained at a monthly auction: the bidders submit pairs of quantity and price/unit ratios: they are willing to provide a capacity of  $x$  Megawatts at a price of  $y$  euros/Megawatt. A supplier may divide his capacity and submit multiple bids, but he has to be able to provide the summed capacity of his accepted bids. The demand of the grid operators is predetermined by past statistics, and the demand is perfectly inelastic due to legal reasons. The auction mechanism is such that capacities with the lowest price/unit ratios are bought until the demand is fulfilled. If the quantity of the last bundle bought exceeds the remaining demand, only the necessary part of the quantity is bought. The auction results are made publicly available, but only the accepted offers. Therefore bidders cannot easily estimate the strategies of the other bidders. However PRC-capable power plants are expensive. The players in the market have good estimates of each others' capacities. Such a market - known demand, few suppliers with approximately known capacities and algorithmic market mechanism - seems ideal for economic analysis. However, it is unclear whether the human decision makers involved introduce too much chaos into this system. In section 2 we present the game theoretical model that has been solved by Kreps and Scheinkman [2]. In section 3 we give up the assumption of rationality and examine models of bounded rationality. In section 4 we discuss the results of econometric models run on time series.

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## 2 Game theoretical model

The Nash-equilibrium solution of a more complicated case is already provided in Kreps and Scheinkman [2]. They discuss the problem of a Bertrand duopoly<sup>1</sup> with capacity constraints and zero production costs, but their findings are easy to extend. Let us assume that there are  $n$  players and the capacity of player  $i$  is  $x_i > 0$ . First let us assume that the 'production' or activating capacity is 0. Let  $D$  denote the demand. If there is no  $j$  such that

$$\sum_{i=1}^n x_i < D + x_j,$$

then the competitive equilibrium of this game is that all firms offer their capacities at 0 price. In a situation when firm were offering their capacities at a positive price there is unavoidably someone who cannot sell his complete capacity. This firm can gain by selling his leftover capacity at half the price, so there is no other equilibrium.

Assume that the activation of capacities is costly, or there is an alternate market for the leftover capacities (which is true in our case: PRC plants can also produce energy at constant rate for long durations) and the per unit costs of the firms are such that  $c_1 \leq c_2 \leq \dots \leq c_n$ . Let  $j$  be the smallest number with

$$\sum_{i=1}^j x_i > D.$$

Then the equilibrium price is  $c_{j+1}$ . In situation with price  $p > c_{j+1}$  there would again be a firm with unsold capacity and he could gain by selling it for  $\frac{p+c_{j+1}}{2}$ .

If there is a firm  $j$  such that

$$\sum_{i=1}^n x_i < D + x_j,$$

then we run into technical difficulties. Without the capacities of firm  $j$ , the demand cannot be satisfied, and demand is inelastic. So firm  $j$  can theoretically ask for *any* price, so there is no optimum price and hence no equilibrium. Obviously this is not the case in the real world. Such behaviour would attract investigations by the authorities for abusing market power. Assuming there is a highest price which is not yet suspicious, the firm  $j$  has to assess whether it is best to sell his partial capacity at the maximum price or his full capacity at competitive price. If he benefits more by selling his partial capacity at maximum price then there is no pure equilibrium as the other firms have no incentive to sell at a much lower price, and a price war of infinitesimal undercutting would ensue. For the mixed equilibrium see Kreps and Scheinkman (1983).

It is worth noting that firms also covertly or tacitly cooperate. Assume there are just two firms, with  $x_1, x_2 > D$  and  $c_1 = c_2 = 0$ . Let the maximum price be normalized to 1. We assume that if the firms bid at the same price the demand is distributed among them equally. As we stated before the Nash-equilibrium would be

<sup>1</sup>A Bertrand duopoly is a market with two suppliers where buyers buy from the supplier with the lowest price until his supply is used up or the demand is fulfilled.

to sell at 0 price. If both firms were to sell at the price of 1 they would both benefit. But this cooperation is not an equilibrium in the one-shot game as a firm can gain by selling at  $1 - \epsilon$  if  $\epsilon$  is a small enough positive number. But if the game is repeated over infinite time periods and future money is discounted at a rate of  $\frac{1}{1+r}$  then there may be infinitely many equilibria. A simple one is constructed using the following strategy: I will price at 1 until the other firm does so as well. If he undercuts me I will start pricing at 0 and will do that forever. This strategy is an equilibrium if undercutting the other is not beneficial, that is there is no  $\epsilon > 0$  such that

$$D(1 - \epsilon) > \frac{D}{2} \left( \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} \right).$$

This is true if

$$r \leq 1$$

which is a reasonable assumption for the interest rate. If there are  $n$  firms, the equation changes to

$$D(1 - \epsilon) > \frac{D}{n} \left( \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} \right)$$

and

$$r \leq \frac{1}{n-1}$$

so cooperation in equilibrium is less likely. The actual data supports this theoretical prediction when the bi-annual auction got replaced by a monthly auction. The effective interest rate dropped, and prices raised by close to 50%. While this can be interpreted as a sign of tacit collusion, it can also mean that the firms had less to lose so they were willing to risk higher bids, and this started an upward trend which encouraged further increasing bid prices.

There were regular price fluctuations on the market, which does not happen in our theoretical model, not even if we consider mixed equilibria. Therefore we decided to give up the assumption of profit maximizing behavior. These models are presented in the next section.

### 3 Models of bounded rationality

In this section we assume that the firms make decisions based on some simple but non-optimal rule. Of course it is impossible to guess what specific rules firms are using, so we decided to invent a few sensible ones and run numerical simulations to see what kind of dynamics evolve.

The model simulates the primary reserves market and uses the same rules for the acceptance of the bids. We simulate some players that have a certain capacity and follow certain strategies for their bids (Table 1). For the strategies the players can use the information of the accepted bids from the previous auctions and their own previous bids. We want to simulate the fact that human players have a random factor to their bidding. Hence, we are interested in the influence of small deviations from the strategies. Therefore we add a factor ( $R$ ) multiplied with a random real number

Ave	Bids the average of accepted bids of the last auction for its total capacity.
Max	Bids the highest of the accepted bids of the last auction for its total capacity.
Avemax	Bids the average of the maximum and the average of the accepted bids of the last auction for its total capacity.
Updown	Always bids total capacity; If the last bid was completely accepted, bids last bid + 5, else bids the lowest bid of the last auction.
Sacupdown	Always bids total capacity; If (part of) the last bid was accepted, bids last bid, else bids the lowest bid of the last auction.
Expol	Extrapolates the highest accepted bid from the last 2 auctions and bids its total capacity on the highest expected accepted bid.
Spread	Splits its capacity in 5 equal parts and bids them with a difference of 10 between the bids; If all the bids are accepted, it replaces the lowest bid with a bid 10 higher than the highest bid last time; If at least 3 bids are declined, it replaces the highest bid with a bid 10 lower than the lowest bid it bid last time; otherwise it will bid the same.

**Table 1:** Strategies

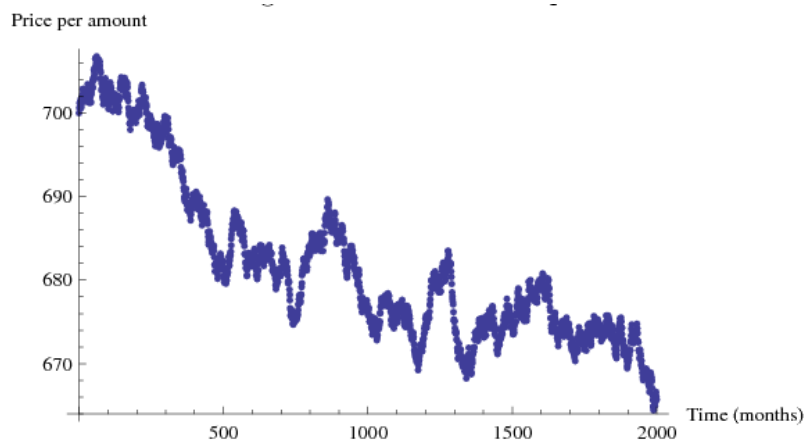
$\epsilon \in [-1, 1]$  to the bids. We simulated 2000 rounds of bidding for a total demand of 80 with 7 players specified in Table 2. We added a random value between -1 and 1 to all bids ( $R = 1$ ).

The maximum accepted bid shows a slow decline (Fig. 1). If we do the simulations without adding a random factor the bidding will converge (results not shown).

If we look at the relative payoffs of the different strategies (the money they made divided by their capacity) we see that Ave and Avemax are doing best (Fig. 2). Sacupdown and Updown are doing almost as well, there is only a difference in payoffs in the beginning (influenced by their initial bid). These are the strategies that do not aim to bet the maximum and always get accepted. Player 6 playing Max had a high capacity and suffered from only getting accepted partially.

Player nr	Capacity	Strategy	First bid (Price, Capacity)
1	15	Sacupdown	(710, 15)
2	5	Ave	(690, 5)
3	15	Avemax	(700, 15)
4	5	Expol	(700, 5)
5	10	Spread	(680, 2) (690, 2) (700, 2) (710, 2) (720, 2)
6	40	Max	(700, 40)
7	30	Updown	(610, 30)

**Table 2:** Capacity and strategy of the players

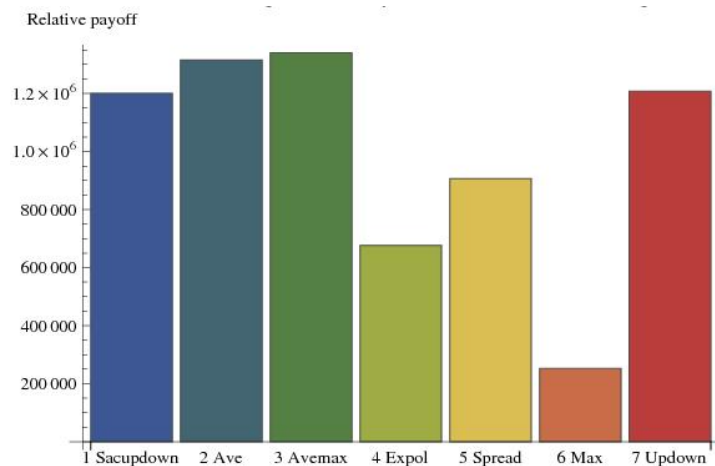


**Figure 1:** Maximum accepted bid over time

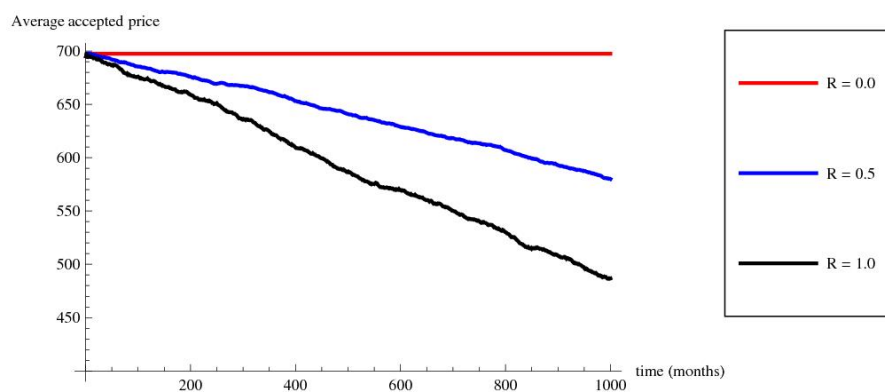
Including some random factor in the bidding will decrease the accepted bids in the auction over time. This is because random lower bids are always seen by the other firms, while random higher bids are sometimes not accepted and therefore not seen by the other firms. Since firms base their strategies on the bids of the last auction they will have a more negative view of the auction if there is a bigger random factor. If all firms follow the Avemax strategy and there is no random factor the market price will stabilize. However, with a random factor the price will decrease (Fig. 3).

We show that the fact that there is some random spread in the bidding will lower the bids in the auction if the firms use a strategy based on the accepted bids of the last auction. In our simulation the strategies that were not too optimistic are doing best. However, all the strategies here are fairly optimistic, if we add strategies that aim for just being accepted (for example bid the minimum bid of the last time), the price would drop very fast. For example, if the sixth firm, playing Max, decided to play Min instead, the price would drop to close to zero within 85 rounds of bidding. Therefore, even though Max was not doing too well, it is doing better than a strategy Min would do in its place. Since firms are looking to maximize their profit they are interested in their payoff and not their relative payoff compared to other firms. So overbidding or withholding capacity might be a good long term strategy. If the capacity is a lot higher, the price will drop fast and if the demand is close to the total capacity the price might even increase. This could justify holding back some capacity because the demand will then be close to the total capacity (or offer at a high price, which is effectively the same, because the other firms won't see the bid). Past data from the German PRC auctions confirms this, see the the huge price increases in November 2008 and April 2011. We don't take the value of the unused capacity into account here. That would also imply a minimum bid by all firms and it could be interesting to include, especially if this value is not the same for all firms. It might also be interesting to include strategies that use their capacity relative to the total demand into account.

We have shown a specific example of a few strategies. However, this shows some general trends such as: the decline caused by random fluctuations, the importance of the total capacity relative to the demand and the effect of a optimistic or pessimistic



**Figure 2:** Payoffs of different strategies



**Figure 3:** Effect of random changes in the bid if all firms play Avemax.

strategy on the general trend of the auction. To generalize these results or use them for a specific case the strategies have to be expanded or calibrated.

## 4 Time series analysis

A natural question was whether time series analysis could yield any promising strategy. Adopting the general attitude of econometric modeling, we assumed that nothing fundamental is known about the data-generating process, therefore our available dataset comprised just the set of accepted bids – pairs of quantities and prices – from December 2007 to January 2011. The question was whether using this data we could find any algorithm that could generate either expectations for the next months, or even a specific bidding strategy that would outperform experts.

First, it was necessary to trim the dataset. Since there was evidence that the data

for the first thirteen months was unreliable for prediction purposes, market conditions having changed vastly within this period – e.g. there were a different number of companies involved, consequently, a smaller overall supply for the primary reserves – we built models using only data starting with January 2009. Notwithstanding our reservations about the length of the resulting time series (just 25 time periods), we attempted to uncover the autoregressive moving average (ARMA) structure of the process, checking whether it could be used for prediction purposes. The ARMA model is a statistical tool that helps to understand the development of a process in time. The model tries to approximate the next point of the time series by taking into account the previous observations and the error terms. For further explanation consult to [1, pp. 43-72]. The advantage of autoregressive models is precisely that they require no theoretical assumptions about the *causal* background of the time series in question, and we thought that this was fitting to our purposes.

The problem of condensing the dataset of observation pairs into a more manageable time series also arose. Consequently, we decided to concentrate on prices, and only take quantities into account as weights for the average price, the latter having obvious implications for bidding policy. For the average only actually accepted quantities were included.

Autoregressive analysis first required us to check whether the series contains unit roots, and to how many levels. For the most common significance level of 5%, the examined process was integrated once. Therefore, the ARMA model was written for the (once) differentiated variable. We have checked for autoregressive and moving average components of a maximum length of 3, giving a total of 16 possible models. The model selection criteria employed was the Schwarz criterion, which penalizes more for model complexity than the Akaike criterion. With this method, we have found that the most fitting model had an MA(3) structure, providing a promising R-squared of 35%, which was relatively robust for changing sample size, popping out as the best model in over half of the cases, including the full sample.

However, despite its relative advantage over alternative ARMA models, the forecasts of an MA(3) regression based on the difference of average price cannot be directly adapted as a bidding strategy. First of all the number of observations is obviously too small, so there is likely an overfitting on data. Moreover, this model cannot be easily interpreted economically: the terms of the equation have both negative and positive signs, and the estimated impact increases with the length of lag, which is counterintuitive. Even if we avoid a specific interpretation, forecasting out-of-data with this model sometimes suggests an average price too close to the actual maximum, which would imply that at least a part of the bid for the primary reserve would not be accepted, potentially leading to great financial losses. Finally, the generated confidence intervals are sometimes wider than the actual spread of prices, indicating that this approach is indeed unreliable.

Although other approaches were tried (including but not limited to changing the model selection criterion, generating different percentiles for the price from the original data and trying to forecast the maximum or the minimum price) econometric analysis could provide no more than *ad hoc* solutions for the specific dataset. These are consequently unusable for out-of-data forecasting. The lack of economic insight even for the best models signaled the theoretical and practical weakness of the method. This suggests that even the most successful model would have been outperformed by

actual expert decisions.

## 5 Conclusion

Our models could explain some phenomena in the PRC auction: the price increase when the time period changed from bi-annual to monthly is due to decreased risk. Occasional price spikes can be explained by the overbidding of a firm with significant market share. While it loses money in the short run, the slowly vanishing price increase might compensate him in the long run. However qualitative explanations of these dynamics do not enable us to give an accurate prediction of tomorrow's prices. Since a few firms have significant market share their bids have a big effect on the price. From data mining we determined that some of the decision makers at these firms use ad-hoc rules when they make their bids, hence foreseeing the equilibrium price perfectly would fall more in the realm of psychology. We advise the bidders to stick close to the minimum price of the previous month. This strategy would have resulted in acceptance of the bid in every month, and it does not give a much lower revenue than a frequently but not always accepted maximum bid would. Prices will fall in the long run, and the cost of keeping them up by withdrawn capacities is too high.

## References

- [1] James D. Hamilton. *Time Series Analysis*. Princeton University Press, 1994.
- [2] D. M. Kreps and J. E. Scheinkman. Quantity precommitment and Bertrand competition yield Cournot outcomes. *The Bell Journal of Economics*, 14(2):326–337, 1983.