Report 2

Optimal Distributed Power Generation Under Network-Load Constraints

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Abstract

In electrical power networks nowadays more and more customers are becoming power-producers, mainly because of the development of novel components for decentralized power generation (solar panels, small wind turbines and heat pumps). This gives rise to the question how many units of each type (solar panel, small wind turbine or central heating power units) can be inserted into any transmission line in the network, such that under given distributions on the typical production and consumption over time, the maximum loads on the lines and components will not be exceeded.

In this paper, we present a linear programming model for maximizing the amount of decentralized power generation while respecting the load limitations of the network. We describe a prototype showing that for an example network the maximization problem can be solved efficiently. We also modeled the case were the power consumption and decentralized power generation are considered as stochastic variables, which is inherently more complex.

2.1 Introduction

Electrical power grids are becoming increasingly complex. The customer used to be solely a power-consumer, whereas nowadays more and more customers are becoming power-producers. Decentralized Power Generation (DPG) refers to an electric power source such as solar, wind or combined heat power (CHP) connected directly to the distribution network or on the customer side of the meter (Pepermans, G. et al., [8]; Chicco and Mancarella, [2]). It has emerged as a key option for promoting energy efficiency and use of renewable sources as an alternative to the traditional generation. Moreover in the near future, decentralized energy buffering is expected to become important, e.g. due to a growth of the electric car market.

These developments pose many questions to grid operators and electricity producers. To what extent is the current power infrastructure suited for the addition of this kind of energy-producing components? Or, at which locations should the infrastructure be extended to handle placements of additional components?

This question is complicated by the fact that the power production of the components strongly varies over time. Different types of components will produce peak power at different points in time, which most likely will differ from the peaks in consumption. Moreover, there are correlations between the yields of multiple components

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of the same type, which are installed at nearby geographical locations. For example, if the sun is shining in a particular street, then it is likely that the sun shines in all streets in the neighborhood.

In many cases, distributed generators can provide lower-cost electricity and higher power reliability and security with fewer environmental consequences than traditional power generators. In contrast to the use of a few large-scale generating stations located far from load centers (the approach used in the traditional electric power paradigm), DPG systems employ numerous, but small plants and can provide power on-site.

Nevertheless, the high complexity of the issues regarding the planning and management of the electric power system and infrastructure for the decentralized power generation calls for powerful analysis tools. One of the most critical factors limiting large scale DPG in an existing network is the possible over-current on connections and over-voltage on nodes. A very large power generation at a moment of low consumption in the grid will usually violate voltage profile constraints. Transmission lines between the low voltage grids may become overloaded due to altered flow patterns resulting from the DPG current contribution. This may require a network reconfiguration or generation limitations on DPG. However, network reconfiguration requires a huge investment for which the distribution network has no incentive as a natural monopoly. Hence, it is important that regulators impose limits on DPG to allow them to participate in the electricity market. In this respect, few papers have addressed the optimal sizing and placement of DPG in an existing distribution network.

Niemi and Lund [7] develop a fast tool to assess and visualize the voltage effects of DPG in an existing distribution network. Using their method, they find that over-voltages with large amount of DPG can be avoided through a proper placement strategy; placing closer to the transformer side will reduce the voltage increase. However, there are quite a few limitations in their method such that it cannot be applied generally. Their static method assumes known load pattern and DPG production over time to predict a modified steady-state voltage profile when introducing DPG, and they believe the dynamic behavior of the electric system can be accessed through a point by point calculation over time. However, in reality, there are high uncertainties in both load and DPG production which makes the net power/consumption more volatile. They restrict their method to a loopless network, because in a loopless network, the cables between adjacent nodes have an unambiguous orientation: the upstream node looks always toward the transformer and downstream node toward the end of the line, so the loopless branched network can be approximated with a single line network by matching downstream consumption and impedance at each node. Nevertheless, it is quite usual that a power distribution network has loops. Moreover, they assume an evenly distributed load along the line and some sort of even distribution of DPG units along the line. This approximation takes into account the voltage differences occurring over transmission line, but not over the individual loads.

Other papers (Gozel and Hocaoglui [3]; Acharya, Mahat and Mithulananthan [1]) propose analytical approaches to calculate the optimal sizing and placement of DPG for minimizing the total power losses in a power distribution system. They document the exact loss formula or loss sensitivity factor for the distribution system.

They examine the effect of size and placement of DPG with respect to loss in the network. However, they only considered voltage constraints and their analyses are based on the power injection or equivalent current injection which they assume to be deterministic. Kuhn and Schultz [6] developed models and algorithms for risk neutral and risk averse power optimization under uncertainty, including a stochastic integer programming model.

KEMA BV addresses many types of questions related to energy networks, and advises grid operators and energy producers. For the SWI we have focused on the following question. Given an existing power grid, we would like to have a method that can quickly determine how many units of each type (solar panel, small wind turbine or central heating power units) can be inserted into any transmission line in the network, such that under given distributions on the typical production and consumption, the maximum loads on the lines and components will not be exceeded. As input, we have used the operating characteristics and statistics of the three types of components and typical usage data.

The transmission of power in each segment of an electrical power network can be determined through a load flow analysis according to Ohm's and Kirchhoff's laws. For this analysis there is standard software available such as Vision Network Analysis[5] for the medium voltage network and Gaia[6] for the low voltage network. This analysis results in a linear relation between the amount of decentralized power generation and the load in the network. We first considered the situation in which the power usage of consumers and the power generated by the decentralized units is assumed to be deterministic, although it can vary over time. We derived a linear programming model for maximizing the amount of decentralized power generation while respecting the load limitations of the network. Linear programming models can be solved quite efficiently by modern solvers, for example CPLEX[7]. We have implemented a prototype for a small example network.

We also modeled the case were the power consumption and decentralized power generation are considered as stochastic variables. This case is inherently more complex, since we have to deal with probabilities of overloads.

The remainder of this paper is organized as follows. In Section 2.2 we study the network model and the load flow analysis. In Sections 2.4, 2.4, and 2.4 we describe the models for the deterministic and stochastic case respectively. Then is Section 2.5 we present numerical experiments for our prototype. Finally, Section 2.6 concludes the paper.

2.2 Network model and load flow

We model the electrical power network as an undirected graph (N,E), where N is the set of nodes and E is the set of edges. A node corresponds to a site of electricity consumption and/or production (e.g. a house with solar panels) or to a connection point. There is an edge between two nodes i and j if there is a cable between the

[5] www.phasetophase.nl
[6] www.phasetophase.nl
[7] http://www-01.ibm.com/software/integration/optimization/cplex/

nodes. We assume that the network is connected, i.e., there is a path between each pair of points in the graph. For the network we define the following entities:

- H set of electricity consumption and/or production points.
- C set of connection points.
- P_i = net power production at node $i \in H$. $P_i < 0$ implies that the power consumption is larger than the power production, $P_i > 0$ implies that production is larger than consumption.
- V_i = voltage at node i
- Q_i = current flowing into or out of the network due to production at i. $Q_i > 0$ means power generation and $Q_i < 0$ means power consumption $i \in H$. For a connection point $i \in C$ we have $Q_i = 0$
- R_{ij} = resistance of cable corresponding to the edge between i and j. If there is no edge between i and j, $R_{ij} = \infty$. Resistance is independent of the direction of current flow $R_{ij} = R_{ji}$.
- I_{ij} = current flowing from nodes i to node j ($I_{ij} > 0$: flow $i \to j$, $I_{ij} < 0$: $j \to i$). Because of this definition, $I_{ij} = -I_{ji}$.

We assume the power at node i is generated at voltage V_i, such that

$$P_i(t) = V_i Q_i(t).$$

We are interested in the behaviour of the local flow I_{ij} given the production and consumption pattern Q_i. The flow of current in the network is governed by the laws of Ohm and Kirchhoff. The voltage drop along network edge (i,j) is given by *Ohm's law*

$$V_j - V_i = R_{ij} I_{ij}. \tag{2.1}$$

Kirchhoff's current law states that the total current entering a node equals the total current leaving it. For node i with net production Q_i this becomes

$$\sum_j I_{ij} = Q_i \tag{2.2}$$

Kirchhoff's voltage law states that the total voltage drop around a closed loop in the network must be zero. Let $L = \{(k_1,k_2),(k_2,k_3),\ldots,(k_n,k_1)\}$ be a closed loop of n nodes. Then we have

$$\sum_{(k_i,k_{i+1}) \in L} R_{k_i,k_{i+1}} I_{k_i,k_{i+1}} = 0, \tag{2.3}$$

where $k_{n+1} = k_1$.

In 1847 Kirchhoff [5] already showed that to determine the current I, it is not necessary to consider equation (2.3) for all cycles, but only for a set of independent cycles. For example if the equation holds for a cycle $\{A, B, C\}$ and a cycle $\{C, B, D\}$, then it also holds for the "sum" $\{A, B, D, C\}$. A well-known method to construct this set of independent cycles in as follows (see also Harary [4]). A tree is a graph without cycles. A spanning tree of a graph is a subgraph which is a tree and contains all nodes. For a connected graph with n nodes a spanning tree has $n-1$ edges.

We take a spanning tree T of the graph. If we extend T by one edge from outside T we obtain a cycle. From the set of edges outside T, we now obtain a set of cycles, where each cycle is obtained by extending T with a single edge. This set of cycles forms a set of independent cycles. In fact it forms a *cycle base*, i.e. a family of cycles which spans all cycles of the graph. Now it is easy to see that the size of a cycle base equals

$$|E| - (|N| - 1).$$

In general, Kirchhoff's voltage law on the elements of a cycle base, implies Kirchhoff's voltage law on all loops. Hence, Kirchhoff's voltage law can be described by $|E| - (|N| - 1)$ equations of type (2.3).

We assume that the local network is connected to an infinite power reservoir and modelled by one node, say ∞, connected to the outside world. This reservoir can provide (or absorb) any amount of net power produced by the local network. For simplicity, we will disregard the voltages and equations (2.1), by assuming that the power at the nodes is produced approximately at a constant voltage. As a result, we can analyse the load flow entirely in terms of currents and resistances. Thus, given the resistance R and the local production Q, we can use (2.2) and (2.3) to calculate I.

For the local flow, we do have to worry about equation (2.2) for the point ∞. We conclude that Kirchhoff's current law can be described by $|N| - 1$ equations of type (2.2). Since Kirchhoff's voltage law can be described by $|E| - (|N| - 1)$ equations of type (2.3), the local flow I on the edges can be expressed in terms of R and Q by $|E|$ equations. From this we can easily show that there is a matrix A such that

$$I = AQ. \tag{2.4}$$

We illustrate this by the following example.

Example. To work with a concrete example, we consider the simple example network of Figure 2.1. This network has five houses indexed $1, \ldots, 5$ with two houses in a closed loop, and three more houses in a radial network. The points with indices a, b and c are connection nodes, that have no generation or usage.

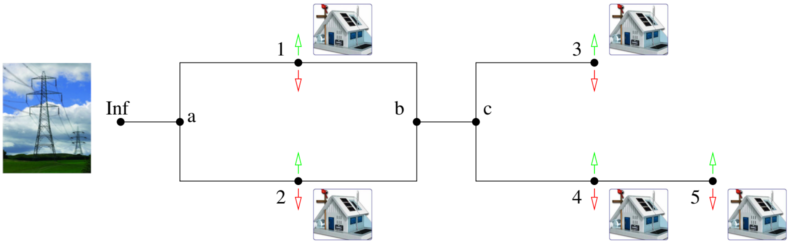

Figure 2.1: Example network

Kirchhoff's current laws for the respective nodes are:

$$
\begin{aligned}
I_{\infty a} - I_{a1} - I_{a2} &= 0 && (a) \\
I_{a1} - I_{1b} + Q_1 &= 0 && (1) \\
I_{a2} - I_{2b} + Q_2 &= 0 && (2) \\
I_{1b} + I_{2b} - I_{bc} &= 0 && (b) \\
I_{bc} - I_{c3} - I_{c4} &= 0 && (c) \\
I_{c3} + Q_3 &= 0 && (3) \\
I_{c4} - I_{45} + Q_4 &= 0 && (4) \\
I_{45} + Q_5 &= 0 && (5)
\end{aligned}
$$

For the loop, Kirchhoff's voltage law gives

$$R_{a1}I_{a1} + R_{1b}I_{1b} - R_{2b}I_{2b} - R_{a2}I_{a2} = 0.$$

For ease of exposition, we assume $R_{a1} = R_{1b} = R_{2b} = R_{a2}$, so that Kirchhoff's voltage law results in $I_{a1} + I_{1b} - I_{2b} - I_{a2} = 0$.

The above equations can be written in matrix form

$$
\begin{bmatrix}
1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{pmatrix}
I_{\infty a} \\ I_{a1} \\ I_{a2} \\ I_{1b} \\ I_{2b} \\ I_{bc} \\ I_{c3} \\ I_{c4} \\ I_{45}
\end{pmatrix}
=
\begin{pmatrix}
0 \\ -Q_1 \\ -Q_2 \\ 0 \\ 0 \\ -Q_3 \\ -Q_4 \\ -Q5 \\ 0
\end{pmatrix}
\tag{2.5}
$$

The matrix on the left, which we denote by B is nonsingular. We also define the injection matrix

$$J=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and the vectors $I=(I_{\infty a},I_{a1},I_{a2},I_{1b},I_{2b},I_{bc},I_{c3},I_{c4},I_{45})^T$ of loads and $Q=(Q_1,Q_2,Q_3,Q_4,Q_5)^T$ of net productions. With this notation, equation (2.5) takes the form

$$BI=JQ.$$

Recall from the above that B is a square nonsingular $|E|\times|E|$ matrix, and we can define $A=B^{-1}J$, cf. (2.4), with

$$A=\frac{1}{4}\begin{bmatrix} -4 & -4 & -4 & -4 & -4 \\ -3 & -1 & -2 & -2 & -2 \\ -1 & -3 & -2 & -2 & -2 \\ 1 & -1 & -2 & -2 & -2 \\ -1 & 1 & -2 & -2 & -2 \\ 0 & 0 & -4 & -4 & -4 \\ 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & -4 & -4 \\ 0 & 0 & 0 & 0 & -4 \end{bmatrix} \tag{2.6}$$

2.3 Local power production and consumption

We write the net production of power at node i as

$$P_i(t)=-U_i(t)+\sum_k S_{ik}(t) \tag{2.7}$$

where U_i is the power consumption and S_{ik} is the generation by a device of type k (e.g. solar panel of given type, wind turbine, CHP unit, ...). Assume we can specify a distribution of local energy sources by choosing constants σ_{ik} such that

$$S_{ik}(t)=\sigma_{ik}\tilde{S}_{ik}(t) \tag{2.8}$$

where $\tilde{S}_{ik}(t)$ are unit production rates (possibly random), e.g. for solar production at a given node. $\tilde{S}$ represents the solar insolation per m^2, multiplied by a (possibly time-dependent) efficiency parameter that incorporates the angle of orientation of the solar panel and its efficiency factor (W/lux). Thus, σ_{ik} determines the size of the production unit (e.g. m^2 solar panels) of type k at node i.

As mentioned earlier, we assume that the power at each node in the local network is produced at a given voltage V_i. We will also assume that these voltages are all approximately equal, i.e. $V_i \approx V_j$. This implies that Q_i depends on σ_{ik} in much the same way as P_i:

$$Q_i(t) = q_{i0}(t) + \sum_k \sigma_{ik} q_{ik}(t) \tag{2.9}$$

2.4 Objective: maximal local energy production under load constraints

Deterministic case

The overall objective is to maximize the collective yearly energy production E by the local energy production units, while obeying constraints on the loads in the local network. In the deterministic setting, these are hard constraints, of the form $|I_{ij}| \leq I_{ij}^m$ with given maximal loads I_{ij}^m. For simplicity, we will not take into account constraints on the voltages (recall that we had assumed the voltages to be given).

We denote the yearly energy production of a unit size device of type k at node i as

$$\Sigma_{ik} = \int_{\text{year}} dt\, \tilde{S}_{ik}(t)$$

The total energy production E is determined by the σ_{ik} and Σ_{ik}. In this section, all Σ_{ik}, and thereby E, are considered to be non-random. The objective is to maximize E under variation of σ. Thus:

$$\begin{aligned} \hat{\sigma} &= \arg\max_{\sigma} E(\sigma) \\ &= \arg\max_{\sigma} \sum_i \sum_k \sigma_{ik} \Sigma_{ik} \qquad (2.10) \\ &= \arg\max_{\sigma} \sigma^T \Sigma \qquad (2.11) \end{aligned}$$

under the constraints

$$\begin{aligned} \forall\, i,j,t: &\quad |I_{ij}| \leq I_{ij}^m \qquad (2.12) \\ \forall\, i,k: &\quad \sigma_{ik} \geq 0 \qquad (2.13) \end{aligned}$$

Because of (2.4) we can write

$$I_{ij}(t) = \sum_{j'} A_{ijj'} Q_{j'}(t)\,. \tag{2.14}$$

By substituting this expression in (2.9) the current constraints (2.12) can be recast as

$$\forall\, i,j,t: \quad -I_{ij}^m \leq A_{ij}^0(t) + \sum_{j',k} \tilde{A}_{ijj'k}(t) \sigma_{j'k} \leq I_{ij}^m \tag{2.15}$$

or

$$-I^m \leq A^0(t) + \tilde{A}(t)\sigma \leq I^m \tag{2.16}$$

with

$$A^0_{ij}(t) = \sum_{j'} A_{ijj'} q_{j'0}(t) \tag{2.17}$$

$$\tilde{A}_{ijj'k}(t) = A_{ijj'} q_{j'k}(t) \tag{2.18}$$

Although the objective function does not depend on time, the constraints do: we need $-I^m \leq A^0(t) + \tilde{A}(t)\sigma \leq I^m$ for all t. Can we find $t_1,...,t_N$ such that if the constraints are satisfied at $t_1,...,t_N$ then they are satisfied for all t? If so, we extend the number of linear constraints (one set for every t_n) and solve the resulting LP.

Summarizing: if we check the constraints (2.15) only at a finite number of points in time ($t_1,...,t_N$), we have to solve the following linear program (LP):

$$\hat{\sigma} = \operatorname*{argmax}_{\sigma} E(\sigma) \tag{2.19}$$

$$E = \sigma^T \Sigma \tag{2.20}$$

$$\sigma \geq 0 \tag{2.21}$$

$$\tilde{A}(t_1)\sigma \leq I^m - A^0(t_1) \tag{2.22}$$

$$-\tilde{A}(t_1)\sigma \leq I^m + A^0(t_1) \tag{2.23}$$

$$\vdots \tag{2.24}$$

$$\tilde{A}(t_N)\sigma \leq I^m - A^0(t_N) \tag{2.25}$$

$$-\tilde{A}(t_N)\sigma \leq I^m + A^0(t_N) \tag{2.26}$$

Benefits of increasing the maximum loads

From the theory of linear programming it is known that each linear program

$$\max\{c^T x | Ax \leq b, x \geq 0\}$$

has a corresponding dual problem

$$\min\{b^T u | A^T u \geq c, u \geq 0\}.$$

The optimal values of the dual variables u are called the shadow prices of the constraints $Ax \leq b$ (i.e., the constraints (2.22) – (2.26) for the network problem considered here). These optimal values can be calculated from the solution of the original (primal) LP. Let $\hat{x}$ and $\hat{u}$ denote the solutions of the primal and dual LP. Assuming these solutions exist, they satisfy $c^T \hat{x} = b^T \hat{u}$. Thus, the shadow prices $\hat{u}$ can be seen as the gradient of the maximum $c^T \hat{x}$ of the (primal) objective function with respect to the constraints b.

Let the shadow prices associated with the constraints (2.22), (2.25), etc. for edge (i,j) be denoted by $\hat{u}_{ij}(t_1)$ and $\hat{u}_{ij}(t_N)$. Similarly, $\hat{w}_{ij}(t_1)$ and $\hat{w}_{ij}(t_N)$ denote the shadow prices associated with (2.23), (2.26), etc. If the I^m_{ij} is increased by a small value ϵ, the value of the maximum yearly energy production $E(\hat{\sigma})$ will increase by

$$\Delta_{ij}(\epsilon) = \epsilon \sum_{n=1}^{N} (\hat{u}_{ij}(t_n) + \hat{w}_{ij}(t_n)).$$

Edges (i,j) for which this value is largest represent connections for which investment in additional load capacity is most beneficial.

Stationary stochastic case

In this section we assume that usage and generation at nodes are stationary random variables, i.e. all U_i and $\tilde{S}_{ik}$ are characterized by probability distributions that are independent of time. For the objective function we take the expectation of the energy production (which, due to the assumption of stationarity, is proportional to the expectation of the power production). That is,

$$\mathbb{E}E(\sigma) = \mathbb{E}\sum_{i,k}\sigma_{ik}\int_{\text{year}} dt\,\tilde{S}_{ik} \tag{2.27}$$

$$\propto \sum_{i,k}\sigma_{ik}\mathbb{E}\tilde{S}_{ik} \tag{2.28}$$

Thus, the objective function is (again) linear in σ.

The constraints must be reformulated. Rather than imposing a hard constraint $|I_{ij}| \leq I^m_{ij}$, we want the probability that currents exceed their threshold to be below a certain level. Thus, we require

$$\text{Prob}(|I_{ij}| > I^m_{ij}) < \epsilon_{ij} \tag{2.29}$$

Alternatively, we can use a single constraint:

$$\text{Prob}(\exists(i,j,)\,|I_{ij}| > I^m_{ij}) < \epsilon \tag{2.30}$$

An interesting, related question is: given a set of constants σ_{ik}, what is the probability distribution for any I_{ij}? That distribution tells us e.g. what the probabilities are for small and large overloads.

Because of the linearity of the system (I depends linearly on Q, Q depends linearly on σ, U and $\tilde{S}$), the random variable I_{ij} is a linear combination of the random variables U_i and $\tilde{S}_{ik}$. We cannot assume that the U_i and $\tilde{S}_{ik}$ are all independent. In fact, for some types of production devices (e.g. solar panels) we expect $\tilde{S}_{ik} \approx c_{ij}\tilde{S}_{jk}$ for any i,j (unless the network is well spread out geographically). In other words, two solar panels of the same size but at different (nearby) locations produce nearly the same power at equal times.

Constructing the probability distributions for the I from those for U and $\tilde{S}$ will be difficult, partly because the dependence discussed above. Another complication stems from the type of distributions for U and $\tilde{S}$: it is questionable that those are close to known distributions (such as Gaussian). Monte-Carlo simulation can provide a way out (but may be time-consuming).

Time-dependent stochastic case

Clearly, it is more realistic to consider U_i and $\tilde{S}_{ik}$ as non-stationary stochastic processes, rather than as stationary random variables. The non-stationarity stems from the dependence of U_i and $\tilde{S}_{ik}$ on seasonality and on the day/night cycle. The objective function, the expected yearly energy production, is still linear in σ:

$$\mathbb{E}E(\sigma) = \mathbb{E}\sum_{i,k}\sigma_{ik}\int_{\text{year}} dt\,\tilde{S}_{ik} \tag{2.31}$$

$$= \sum_{i,k}\sigma_{ik}\int_{\text{year}} dt\,\mathbb{E}\tilde{S}_{ik}\,, \tag{2.32}$$

where we have used $\mathbb{E}\int dt\,\tilde{S} = \int dt\mathbb{E}\tilde{S}$ because all $\tilde{S}_{ik}$ are non-negative. Formulating (or estimating) suitable stochastic processes will be a major challenge. In fact, a hybrid approach (consisting of deterministic signals incorporating the daily, weekly and seasonal cycles, and supplemented by a stationary noise) may be plausible.

2.5 Computational experiments

For this paper we conducted numerical experiments for the deterministic case of Section 2.4 only. There was insufficient data for testing the stochastic model.

For the SWI, typical solar production and household usage data were provided by KEMA in the following form: 1) a database containing the instantaneous power flow in Watts for 27 households at 10 minute intervals for one week and 2) solar insolation data in lux at 10 minute intervals for one year.

Making use of the network of Figure 2.1 and corresponding matrix A (2.6) we solved the deterministic LP (2.19)–(2.26) on a one week interval with time constraint period $\Delta t = t_{n+1} - t_n = 10$ minutes. The usage data $U_i(t)$ was taken from the first 5 households of the provided data.

We took all power line maximum load constraints to be $I^m_{ij} = 70A$. We assume only a single type of decentralized generation, namely solar energy. To this end we ignore the second index k on source terms and denote them simply by $S_i(t)$, etc. We chose $S_i(t) = (100\ \text{W/m}^2)\tilde{S}_i(t)\sigma_i$, so that σ_i can be interpreted as the surface area of solar panels in m^2 at node i. Solar insolation $\tilde{S}_i$ was taken from the first week of the given datafile, and assumed to be uniform over the model neighborhood.

The solution of the optimization problem is shown in Figure 2.2. For the optimal configuration, Figure 2.2a shows the loads in Amperes on all edges. Loads a1 and a2 are approximately equal. The critical load is reached on edges a1 and a2 after approximately 6.5 days.

The optimal configuration of solar panels is

$$\begin{aligned}\sigma_1 &= 122\ \text{m}^2\\ \sigma_2 &= 121\ \text{m}^2\\ \sigma_3 &= 31\ \text{m}^2\\ \sigma_4 &= 24\ \text{m}^2\\ \sigma_5 &= 8\ \text{m}^2\end{aligned}$$

For this arrangement, the production in kW at each household is shown in Figure 2.2b. Productions at nodes 1 and 2 are nearly equal and significantly greater than

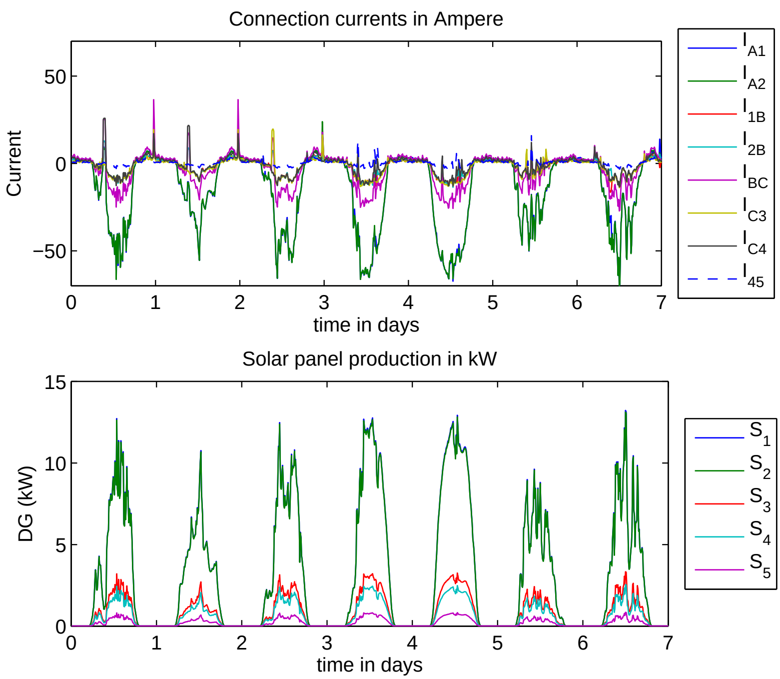

Figure 2.2: Optimal configuration of solar panels: (top) current flow through each network edge for the optimal configuration (negative flow is upstream, consistent with a net surplus in decentralized generation); (bottom) total solar generation at each household node for the optimal configuration.

nodes 3–5. The total optimal production is 1340 kWh/yr, with a net positive energy production of 1070 kWh/yr.

This fact hints at a possible problem with the simple optimization model used here. In particular, the benefit to a consumer of placing solar panels is dependent on that consumer's node in the network topology. For an optimal production, some consumers will gain a much more significant advantage than others. We also computed an alternative configuration for a strict 'fair play' scenario in which we assume all households are allowed an equal maximum solar production, enforced by taking $\sigma_i = \sigma = const$. We manually iterated to obtain an approximate best value of σ of 50 m^2. Figure 2.3a shows the loads on each network edge. The critical load again occurs on edges a1 and a2 after 6.5 days. In Figure 2.3b the production of all households is equal by assumption. Under the 'fair play' scenario, the total production is reduced to 1100 kWh/yr and net production to 830 kWh/yr.

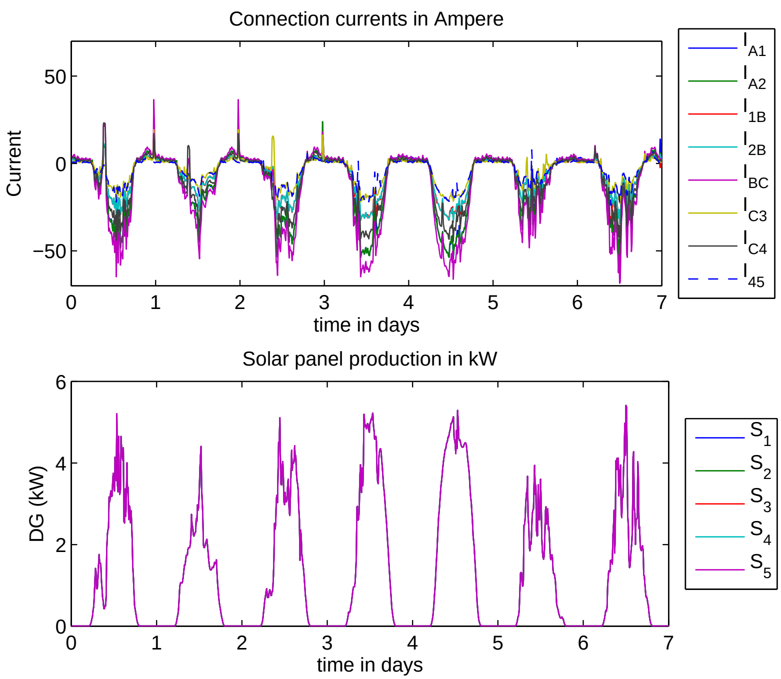

Figure 2.3: Optimal 'fair play' configuration of solar panels: (top) current flow through each network edge for this configuration; (bottom) total solar generation at each household node is equal.

2.6 Conclusion

In this paper, we developed a method that can quickly determine how many units of each type (solar panel, small wind turbine or central heating power units) can be inserted into any transmission line in the network, such that under given distributions on the typical production and consumption, the maximum loads on the lines and components will not be exceeded.

We first considered the situation were the power production and consumption are considered deterministic but vary over time. We derived a linear programming model for maximizing the amount of decentralized power generation while respecting the load limitations of the network. Since linear programming problems can be solved efficiently this is a promising result from the viewpoint of the application. We presented an initial model for the case where power consumption and production are considered as stochastic variables.

For the deterministic case we implemented a prototype in Matlab for a small example. The results are promising since we could quickly compute the optimal allocation of power generation units with a 10 minute time granularity. The results revealed that the optimal allocation is unbalanced in the sense that houses closer to the connection point to the high voltage network are allowed to generate much more power that house located further from this connection, consistent with the findings of Niemi and Lund [7]. To achieve complete fairness, we tested the situation were each house generates the same amount of power. Then the financial benefits are more uniformly distributed among the consumers. However, this provides a significantly lower power production. Consequently, the development of intelligent fairness criteria, which for example can be achieved by adding additional constraints to the linear programming model, is an interesting issue for further research.

2.7 References

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