3 Math Fights Flooding

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Abstract

Due to climate changes that are expected in the coming years, the characteristics of the rainfall will change. This can potentially cause flooding or have negative influences on agriculture and nature. In this research, we study the effects of this change in rainfall and investigate what can be done to reduce the undesirable consequences of these changes.

Keywords: climate change, rainfall, drainage system

3.1 Introduction

At the 2008 Study Group Mathematics with Industry one of the problems concerned the impact of climate change on Dutch water management practices. More specifically, we were asked to study the effect of the increasing intensity of peaks of precipitation events on the water system managed by “het Waterschap Regge en Dinkel”. Some explanation of the nature of this problem owner is in order. A Dutch “waterschap” is an institution run by a democratically elected board that is in charge of the management of the water quantity and quality of open water (streams, brooks, lakes, ditches and canals) in a given region. The board is elected by the local inhabitants and the institution is self financing: it determines the level of certain local taxes and collects those taxes for its own use. One of its main tasks is to protect the inhabitants against flooding and to manage the water levels such that agriculture, nature and shipping are supported. In the remainder of this paper we will use the term “water board” as a rough translation of “waterschap”.

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Figure 3.1: Twente (source: Waterschap Regge en Dinkel).

The Water board Regge and Dinkel is in charge of an area of approximately 40 by 40 kilometers containing the towns of Almelo, Enschede and Hengelo (Figure 3.1).

The problem statement limited the area of interest to the area that, due to terrain elevation and hydrology, discharges its precipitation into the stream the Regge. This area is called the *catchment* of the Regge. The Regge in its turn discharges into the river Vecht.

We examined the Sobek[6] model that was made available by the water board and found that the region below the Twente Kanaal discharges mostly into the Twente Kanaal despite the presence of culverts under the Twente Kanaal. This provided a clear southern border for the catchment. The total Regge catchment consists of a considerable number of subcatchments. A subcatchment is a subarea that discharges all its water via one point on its boundary into a small stream or canal.

In brief, the problem is to find a way to design and evaluate adaptations of the Regge catchment that will keep the discharge peak into the Vecht within a given envelope. Of course, this discharge peak varies in time. To establish general recom-

[6]Trademark of WL — Delft Hydraulics (part of Deltares)

mendations, the water board agreed on defining a typical precipitation event, which serves as a kind of benchmark for any system. This is simulated under the assumption of uniform rainfall over the catchment. This standard precipitation event is a 10-day period of rainfall data (preceded by a long period of almost 40 days with a constant minimal amount of rain to counter initialization effects in a model such as Sobek) as shown in Figure 3.2. For each subcatchment area this precipitation event will lead to a discharge curve that lags behind the precipitation curve and is longer than 10 days. Examples of such discharge curves are shown in Figure 3.3.

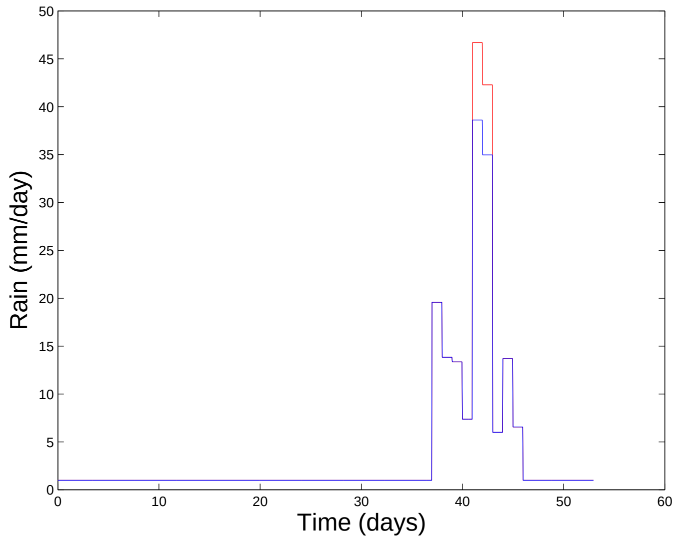

Figure 3.2: Benchmark precipitation event before (blue) and after (red) after climate change.

The discharges from different subcatchments flow together in the Regge. The water involved arrives at the Regge with a time delay that is mainly determined by the distance between the discharge point of the subcatchment under consideration and the Regge. The discharges from all the subcatchments sum up with the appropriate time delays in the Regge. In turn, the Regge discharges its water into the Vecht and a typical Regge discharge curve for the benchmark precipitation event in the current climate is the blue curve in Figure 3.4. This discharge has been computed using the Sobek model. In this figure, the red curve is the maximal discharge imposed to us by the Water Board Regge and Dinkel. The discharge curve is obtained when the standard precipitation event, which is a kind of worst-case rainfall in the current climate, is applied to the present situation in the Regge catchment. It is important to preserve the dip in the discharge after 46 days to allow for the discharge peak from another catchment that flows into the Vecht further upstream. This is an important boundary condition for the study of this project. After the climate change the response to the *new* standard precipitation event, which is a kind of worst-case rainfall in the future climate should respect the upper bound in the discharge curve indicated in red. However, as shown in Figure 3.5, without additional measures,

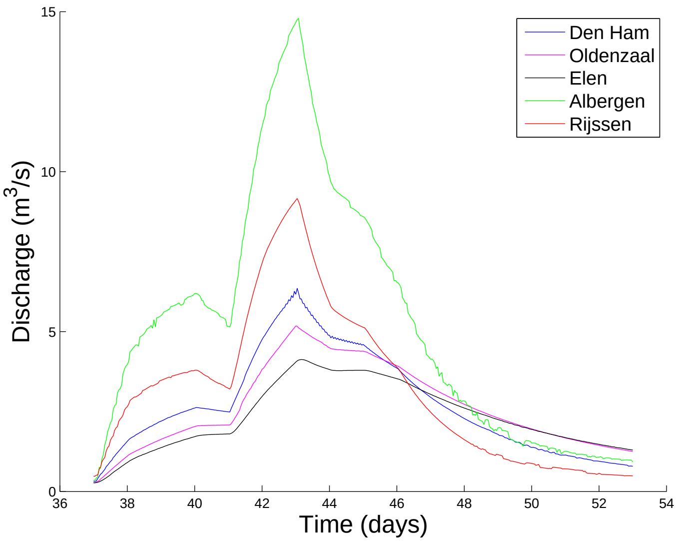

Figure 3.3: Discharge of selected catchments.

the expected discharge (indicated in blue) clearly violates this upper bound. The objective of this study is to look at measures that can be taken such that we get for instance a discharge as indicated in green which mostly respects the given upper bounds.

In other words, the aim of this project is to study what happens if the rainfall would intensify due to climate change. To show the effect we artificially increased the peak discharge in the standard precipitation event in such a way that the total volume in the event increased by ten percent (see Figure 3.2).

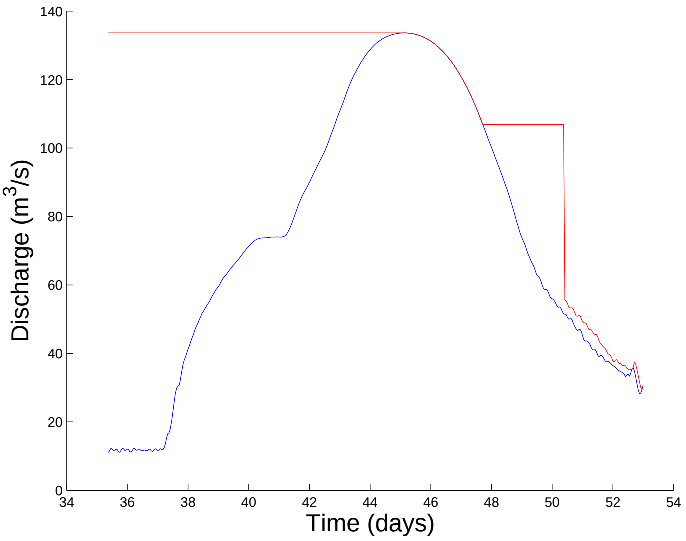

Figure 3.4: Discharge in current climate (blue) and maximal discharge (red).

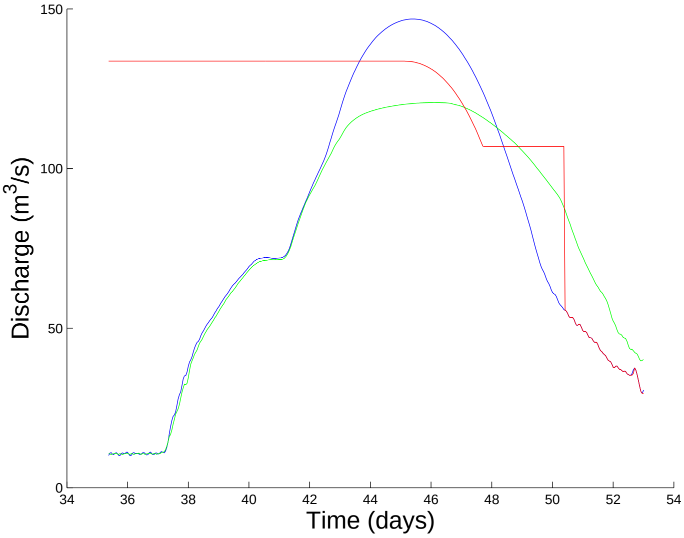

Figure 3.5: Discharge after climate change without and with additional measures together with maximal discharge.

To avoid undesirable discharge rates of the Regge due to increased rain fall, the water board suggested the following possible measures as viable components for a solution.

- *Improved drainage in a subcatchment.* This results in an earlier release, and in a narrower discharge peak from that subcatchment. The improvement could be achieved by additional drainage pipes and/or drainage ditches. However, this can also be realized, to a certain extent, by lowering of the overflow heights of the weirs. Earlier arrival of the run-off at the Vecht from a certain subcatchment could reduce the height of the peak by a better spreading over time of the discharge of the different catchments over time.

- *Slower drainage in a subcatchment.* This results in a later release and a flatter discharge peak from the subcatchment. Reduction of the drainage can be achieved by removal of drainage pipes and/or drainage ditches or by raising the water level in the drainage ditch network. This can also, to a certain extent, be realized by increasing the overflow heights of the weirs. This increases the available storage in the soil and the local collection canals. It flattens and delays the entry of the discharge peak from this subcatchment into the transport canals. Later arrival of a flattened discharge peak can reduce the height of the total discharge peak arriving at the Vecht directly by the flattened peak of the discharge of the subcatchments or, indirectly, by a better spreading over time of the discharge of the different catchments.

- *Storage.* Adding storage basins has effects that are similar to those of slowing the drainage of a subcatchment, but they are more flexible as they can also be used to flatten and/or delay a discharge peak that has already left the soil and the collection canals of a subcatchment. It can affect peaks in the transport canals. Again, as argued before, later arrival of a flattened discharge peak can reduce the height of the peak arriving at the Vecht.

For the total discharge into the Vecht, we must take all contributions of the subcatchments into account. If we link subcatchments whose discharge peak reaches the Vecht at approximately the same time, we get a set of isochrones on the map. This illustrates two aspects of the problem. First, for narrow peaks the longest isochrone (the line connecting points from which water will take the same amount of time to reach the discharge point into the Vecht) will tend to dominate the discharge peak. Second, for wide peaks or rainfall-runoff curves with fat tails the later peaks will piggy back on top of the earlier ones and dominate the discharge peak. The second process will later be confirmed by a sensitivity analysis. The scale of the area (about $40 \times 40\,\mathrm{km}^2$), combined with the width of the peaks from separate subcatchments and the average transport velocity of $1\,\mathrm{m/s} = 86.4\,\mathrm{km/day}$ (according to Regge and Dinkel) implies that the first process does not play a role of much importance.

In Section 3.2 we will obtain a simple model for the discharge based on fitting the data provided to us by the very detailed Sobek model. In Section 3.3 we will model one meadow with adjacent ditches in detail. It will be shown that this model, after suitable fitting of the physical parameters, fits very closely to the earlier model even for an area of more than 1000 hectare which has a lot of detailed structure (small ditches; non-uniform soil characteristics, etc) which are not taken into account in the physical model. In Section 3.4, the sensitivity of the discharge curve in the Vecht to changes in the parameters of the model is analyzed for a specific subcatchment. This gives an idea what can be done to modify the discharge into the Vecht by taking specific actions in suitably chosen subcatchments. Resulting recommendations of our analysis are presented in Section 3.5.

3.2 A dynamical relation between precipitation and discharge

In this section we develop a dynamic model to relate a known discharge curve of a subcatchment to a known precipitation curve, see [1]. In the next section we shall outline how a physical model for the discharge curve of a subcatchment can be obtained which, for a given rainfall data, will result in a discharge curve. The latter curve clearly still depends on certain physical parameters used in the model. In contrast, in this section both the rainfall and discharge curves are given and then a dynamic relationship is fitted between the two curves.

In a subcatchment C, we have during day i an amount of rainfall r_i, which leads to a total discharge d_i in m^3 over that day into the release point of the subcatchment.

Here r_i is the amount of m^3 of rainfall which is the product of the rainfall in a particular day (indicated in Figure 3.2) times the area of the subcatchment (we assume uniform rainfall over the whole region).

A transition is introduced that quantifies on day i the fraction ρ of r_i that is discharged and a fraction $1-\rho$ of r_i that is kept within the catchment. The dynamics within one day are discarded. That means that in the first day, i.e. at the start of the rain event, a fraction ρ of the rainfall r_1 is discharged, and a fraction $1-\rho$ is kept in the catchment. To initialize the model we assume there is no water in the catchment at the beginning of this event. At the next day, the discharge d_2 is given by:

$$d_2 = \rho r_2 + \rho(1-\rho)r_1,$$

where a fraction ρ of the new rainfall is discharged but also a fraction ρ is discharged of the remaining water in the system due to rainfall of earlier days. For a specific subcatchment we have observations of rainfall and discharge over n days and we obtain:

$$d_{i+1} = (1-\rho)d_i + \rho r_{i+1}, \qquad d_0 = 0. \tag{3.1}$$

This can alternatively be presented using a matrix representation:

$$\begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{pmatrix} = \begin{pmatrix} \rho & 0 & \cdots & 0 \\ (1-\rho)\rho & \rho & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ (1-\rho)^{n-1}\rho & \cdots & (1-\rho)\rho & \rho \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{pmatrix} =: A \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{pmatrix}. \tag{3.2}$$

Hence, the discharge has been approximated by a one parameter model. This parameter, however, is specific for each subcatchment. It is governed by the physical conditions of the catchment, like the lateral movement, the vertical changes in elevation, the carriage capacity of the soil and the physical soil unit composition. The parameter indicates in an averaged way how fast the rain is discharged into the canal system outside the area.

3.2.1 Estimation

Estimation of parameter ρ was carried out by a least squares method. Using (3.1), we first note that the matrix in (3.2) has an inverse with a nice structure and we obtain:

$$A^{-1} = \rho^{-1} \begin{pmatrix} 1 & 0 & \cdots & \cdots & 0 \\ \rho-1 & 1 & \ddots & & \vdots \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \rho-1 & 1 \end{pmatrix}$$

For several subcatchment we compared the actual discharge from the Sobek model to the discharge predicted by our model. The expression

$$\left\| \rho A^{-1} \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{pmatrix} - \rho \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{pmatrix} \right\|^2 ,$$

which is is equivalent to:

$$\sum_{i=0}^{n} \|d_{i+1} - (1-\rho)d_i - \rho r_{i+1}\|^2$$

is then a quadratic function in ρ and the minimization of this error to find the optimal value for ρ is then easily achieved. The first 35 days of the rainfall in 3.2 are intended to reduce the effect of initialization. This is crucial in the Sobek model. In our case, the initialization is only related to setting $d_0 = 0$. However, our model needs to be more accurate in days where the discharge is substantial. We improved this process slightly by scaling the squared error by the actual discharge per day:

$$\sum_{i=0}^{n} \|d_{i+1}\| \|d_{i+1} - (1-\rho)d_i - \rho r_{i+1}\|^2$$

This weighting makes the model more accurate during days with a large discharge.

3.2.2 Results

Area	$\hat{\rho}$
Elen	0.15
Oldenzaal	0.16
Den Ham	0.27
Albergen	0.39
Rijssen	0.40

Table 3.1: Estimated ρ coefficients for 5 selected catchments.

We obtained the results listed in Table 3.1 for five selected catchments. Rijssen and Albergen have the largest values of ρ which corresponds to a high peak and a short tail, since most of the rain is discharged into the canal system within a few days. This is clearly consistent with the discharge curves in Figure 3.3. Elen and Oldenzaal have a low value of ρ and hence a low peak and a long tail. These areas keep the rain within the catchments and slowly discharge it into the canal system.

These results are as expected since the Rijssen catchment is located on sandy soil on a large elevation and, hence, the catchment will have a smaller carrying capacity than the other catchments. Consequently, the discharge occurs in a shorter period.

3.3 Rake model

The model proposed in the previous section uses a simple model ignoring for instance the faster dynamics during the day but also ignoring the spatial structure within a subcatchment. In this section, we propose a simplified one-dimensional ground water and hydraulic model to investigate an optimization strategy for designing catchment basins and ground water level management. It incorporates explicitly the weirs and the spatial structure and hence can be used to study the effects of raising or lowering the overflow heights of the weirs or the introduction of additional ditches. It is called the “rake model” because the river Regge is assumed to be connected to a series of ditches associated with two adjacent meadows. Rain will uniformly fall on the whole region, thus also on each meadow. A simple one-dimensional diffusion model is set-up to manage the transport of rain water into the ground to an adjacent ditch. Each (half) meadow is connected to a ditch. Each ditch runs into the Regge and is controlled by a weir at its exit point. And, finally, this exit point has a certain distance to the mouth of the Regge into the river Vecht. Each meadow is chosen to be rectangular and has a width W and length L, the latter also being the length of the ditch. See also Figure 3.7.

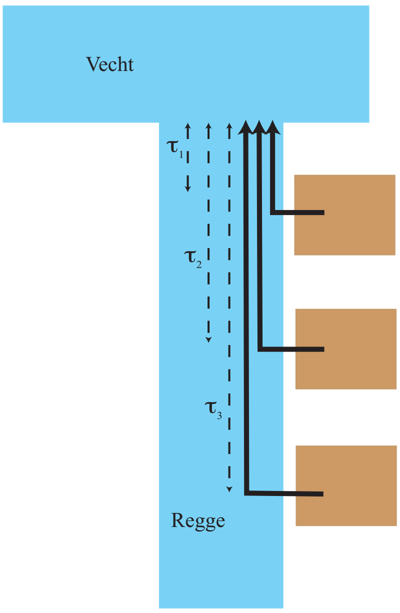

Figure 3.6: Sketch of subcatchments with a different distance to the Vecht, leading to a time lag τ_i in the time when the water reaches the Vecht.

We consider $m = 1, \ldots, M$ meadows and consider one meadow-ditch combination or catchment with index m, dropping the index m at first for ease of notation. Rain water seeps into the ditch from the meadow and the ground water level $h = h(x,t)$ in the pasture depends on the distance x from the ditch with $x \in [0, W/2]$, and time t. The ditch lies at $x = 0$ and the middle of the meadow at

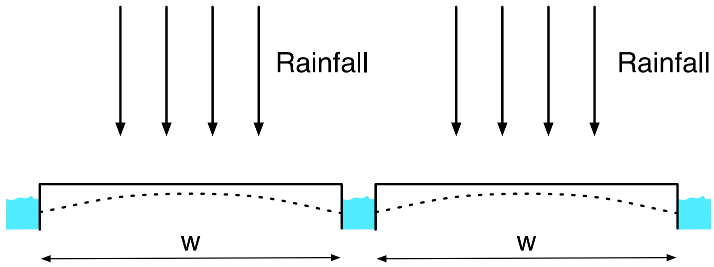

Figure 3.7: Cross section of a meadow with a ditch on each side. The water level is also indicated.

$x = W/2$. Diffusion with diffusion constant μ and soil permeability k governs the dynamics as well as the rain fall $R = R(t)$. The ground water level is assumed uniform in the direction along the ditch; hence we ignore end effects. The governing equation is

$$\frac{\partial h}{\partial t} = \mu \frac{\partial^2 h}{\partial x^2} + \frac{R}{\varphi} \tag{3.3}$$

with φ the porosity of the soil. The water level $h_0 = h_0(t)$ in the ditch is as follows

$$\frac{\mathrm{d}h_0}{\mathrm{d}t}(t) = \frac{\varphi \mu k}{b}[h(0,t) - h_0(t)] - \frac{\sqrt{2g}}{L}\max(h_0 - h_w, 0)^{3/2}, \tag{3.4}$$

in which k is a permeability coefficient, g is the acceleration of gravity, and the last term models a weir at the entrance of the ditch into the Regge. The last term consists of a standard hydraulic approximation for flow over weirs, see [4]. The height of the weir $h_w = h_w(t)$ is a specified function (of time); it can be used to control the outflow of water into the Regge hydraulic system. Catchment basins are modeled simply by specifying a different width $b = b(t)$ of the ditch; it is also a specified function of time. The boundary conditions involve symmetry at $x = W/2$, and consistency at $x = 0$:

$$\frac{\partial h}{\partial x}(0,t) = k[h(0,t) - h_0(t)] \qquad \text{and} \qquad \frac{\partial h}{\partial x}(W/2,t) = 0. \tag{3.5}$$

It is useful to consider the volume balances of water. The change in time of the volume $V = V(t)$ of water in the meadow, associated with one ditch, follows by integration of the diffusion equation (3.3) over the relevant area $W/2 \times L$ and multiplication by φ, while using the boundary conditions (3.5); we obtain

$$\frac{dV}{dt} = \varphi\, L\, \frac{d}{dt}\int_0^{W/2} h(x,t)\, dx = -\mu\, k\, \varphi\, L\, \big(h(0,t) - h_0(t)\big) + \tfrac{1}{2}RLW. \tag{3.6}$$

The change of volume V_0 of water in the ditch follows by multiplication of (3.4) with $L\,b$, to obtain

$$\begin{aligned} \frac{dV_0}{dt} &= b\,L\,\frac{d}{dt}h_0(t) \\ &= \mu\,k\,\varphi\,L\,\big(h(0,t)-h_0(t)\big) - \sqrt{2\,g}\,b\,\max(h_0(t)-h_w(t),0)^{3/2}. \end{aligned} \tag{3.7}$$

Hence, we observe that the discharge from the meadow into the ditch is consistently modeled as

$$\mu\,k\,\varphi\,L\,\big(h(0,t)-h_0(t)\big).$$

The total discharge $Q = Q(t)$ of the ditch over the weir and into the river Regge follows from (3.7) as

$$Q(t) = \sqrt{2\,g}\,b\,\max(h_0(t)-h_w(t),0)^{3/2}. \tag{3.8}$$

We use $Q_m(t)$, instead of $Q(t)$, to indicate the discharge of ditch-meadow combination number m into the Regge which lies at a distance D_m from the mouth of the Regge into the Vecht. It is assumed that water released into the Regge from a ditch flows with a constant velocity v to the Vecht. Hence, water released from ditches of meadows lying further away from the Vecht will travel longer. We immediately see an optimization strategy emerge: by delaying or accelerating fallen rain water to reach the Regge as a function of the location of the meadow from the Vecht we may be able to avoid flooding downstream at the Vecht. Hence, the maximum discharge of water into the Vecht may be managed.

3.3.1 Numerical discretization

To facilitate the numerical discretization, we used a non-dimensional form of the model (3.3)–(3.5). These non-dimensional equations have subsequently been discretized with a finite difference methods, second order in space and first order in time. An explicit forward Euler time discretization is used for the diffusion equation, and the water level equation (3.4) is discretized semi-implicitly by integrating $h_0 - h_w$ instead of h_0 and splitting the nonlinear term as $\sqrt{(h_0^n - h_w^n)}\,(h_0^{n+1} - h_w^{n+1})$ with current time level h_0^n and future time level h_0^{n+1}, and so forth. A time step restriction follows directly from a maximum principle. We refer to a standard text book on numerical methods, see [3].

3.3.2 Numerical results

For simplicity we took a square meadow, i.e. $L = W/2$ and let rainwater, fallen on a meadow of area L^2, seep diffusively into one ditch. Firstly, we gauged the parameters μ, k and φ based on a reference simulation of the Sobek model. The

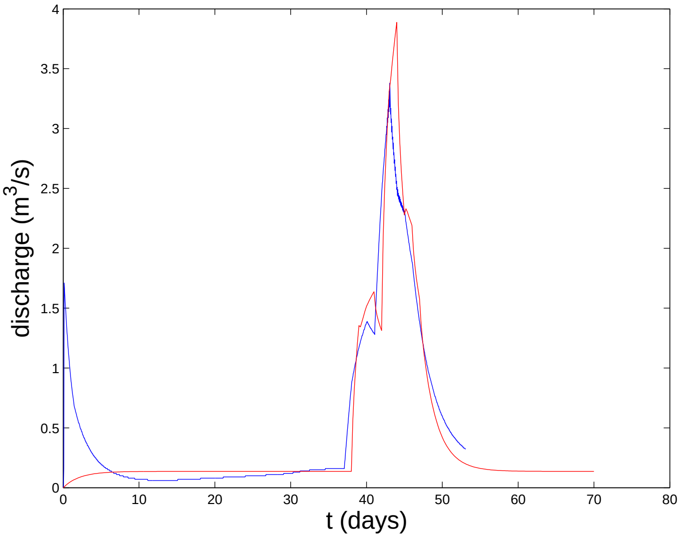

Figure 3.8: Comparison of the Sobek model (blue) and rake model (red) for an area near Den Ham.

Sobek model was run with the heavy ten-day rainfall distribution shown in Fig. 3.2. Subsequently, the discharge of a catchment area located between Den Ham and Vroomshoop concerning an area of $A_r = 118 \times 10^5\,\mathrm{m}^3$ was taken. We performed a run for one meadow of size $L^2 = 200\,\mathrm{m}^2$ scaled with factor s_f such that $L^2 s_f = A_r$, and compared the run-off curves. For the values $\mu = 4\,L_s^2/T$ and $k = 10/L_s$ and spatial scale $L_s = 50\,\mathrm{m}$ and time scale $T = 1$ day, the agreement between the Sobek model and one meadow in our rake model is surprisingly good, see Fig. 3.8. Other parameter values are $b = 2\,\mathrm{m}$, $h_w = 0.5\,\mathrm{m}$, and initially we filled the ditch to weir level, e.g., using initial conditions $h_0(0) = h_w$, and also $h(x,0) = h_0(0)$. Or, perhaps more appropriately, we note that the model is rainfall driven, and the sensitivities on μ and k appear to be relatively small.

Secondly, we considered the rake model with three meadows and ditches, at distances $D_m = m\,L_d$ with $m = 1,2,3$ away from the Vecht. We took $L_d = 20 \times 10^3\,\mathrm{m} = 20\,\mathrm{km}$ and the flow velocity v was taken to be $v = 1\mathrm{m/s}$. The (imaginary) water board for the Vecht has given us a maximum discharge rate of $8\,\mathrm{m}^3/\mathrm{s}$ of Regge water that is allowed to flow into the Vecht. In the base run the three ditches have the same parameter values as above, the only difference being their distance to the river Vecht. Our simulations for the same rainfall as in Figure 3.2 then show that the discharge peaks of each catchment arrives with a delay of about a quarter day $(20 \times 10^3/(3600 \times 24)$ day) into the Vecht, see the lines for the three shifted peaks of about discharge heights $4\,\mathrm{m}^3/\mathrm{s}$ in Figure 3.9. The accumulated discharge of these three catchment supersedes the allowed discharge maximum denoted by the fat horizontal line approximately between days 42 and 46. In our first attempt to optimize, we increased the weir height in the last catchment area to $0.65\,\mathrm{m}$, while starting the ditch level at $0.5\,\mathrm{m}$. Hence, the ditch of length A_r first needs to be filled

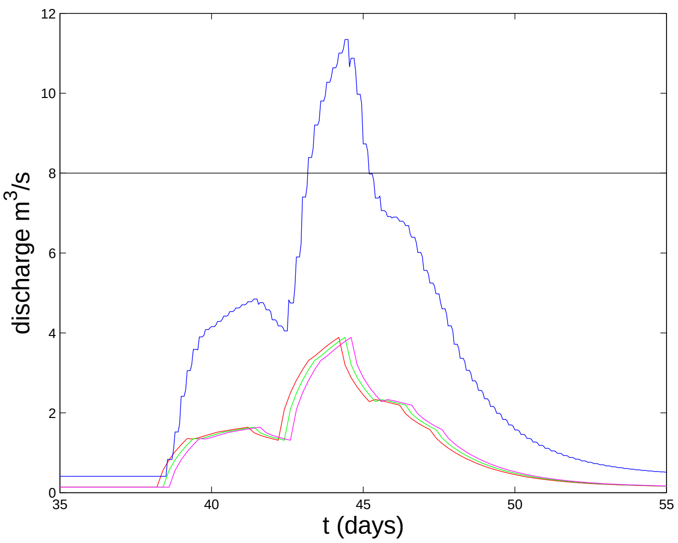

Figure 3.9: Discharge rates of the three catchment areas and their accumulated values (blue).

before rain water flows into the Regge. This constitutes a delay. In Figure 3.10, we see that the discharge peak of the third catchment area (indicated in magenta) starts later, at day 42 instead of day 38 as we saw in the first run, but the accumulated discharge denoted by the blue line, is still too high. Flooding thus still occurs. Our final strategy, in Figure 3.11, is to heighten the weir to 0.6 m and lower the water level in the ditch and the meadow to $h_0(0) = h(x, 0) = 0.25$ m, for example, by an early precautionary release of water. This mimics the use of an additional storage basin. As a consequence, the discharge peak (in magenta) in the lower right half of the plot, is greatly reduced, and assures that the accumulated water discharge of Regge water into the Vecht stays below the maximum discharge level. Clearly, these changes need to be optimized but this can only be done if other factors are taken into account. For instant, increasing or decreasing the overflow level of a weir has economic effects on agriculture in the region, has ecological effects, et cetera. Also zoning plans might not allow certain actions to be taken.

3.4 Sensitivity analysis

In this section we investigate the influence of measures taken in individual subcatchments on the discharge curve of the Regge $D(t)$. The latter is the sum of the discharge curves of individual subcatchments $D_m(t)$, $m = 1, \cdots, M$ in the following way

$$D(t; \rho_1, \cdots, M) = \sum_{m=1}^{M} D_m(t - \tau_i; \rho_i). \tag{3.9}$$

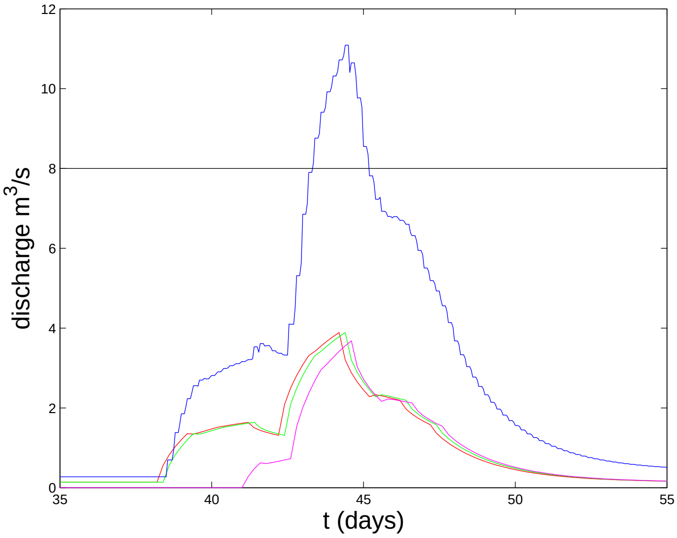

Figure 3.10: Discharge rates with weir height 0.65 m and initial ditch level 0.5 m for one subcatchment (red).

Here, τ_m is the time lag resulting from the fact that the water from a subcatchment needs to flow from the exit point of that subcatchment to the point where the Regge discharges into the Vecht, see Figure 3.6. As discussed in Section 3.2, each individual discharge curve D_m can quite accurately be characterized by only one parameter, ρ_m. From the simple structure of (3.9) it directly follows that

$$\frac{\partial D(t;\rho_1,\cdots,M)}{\partial \rho_m} = \frac{\partial D_m(t-\tau_m;\rho_m)}{\partial \rho_m}. \tag{3.10}$$

In Section 3.2 we introduced discretized versions d_i (indicating the discharge during day i) of $D_m(t;\rho_m)$ (indicating the discharge at time t). In that representation the derivative with respect to ρ_m can for any m be explicitly indicated as:

$$\frac{\partial}{\partial \rho_m}\begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{pmatrix} = \begin{pmatrix} 1 & 0 & \dots & 0 \\ (1-2\rho_m) & 1 & \cdots & 0 \\ \vdots & \dots & \dots & \vdots \\ (1-\rho_m)^{n-2}(1-n\rho_m) & \dots & (1-2\rho_m) & 1 \end{pmatrix}\begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{pmatrix}. \tag{3.11}$$

So, given the value ρ_m of a subcatchment and given a standard (or adjusted) precipitation curve, the derivative curve $\partial D_m/\partial \rho_m$ is easily approximated as a function of time. An example is given in Figure 3.12.

This curve gives an indication of the sensitivity of any discharge curve to changes in the corresponding ρ. From this figure it is clear that the effect of a ρ is largest about 6 days after the rainfall started. This strongly coincides with the peak positions in both the precipitation and discharge peaks. The conclusions from such

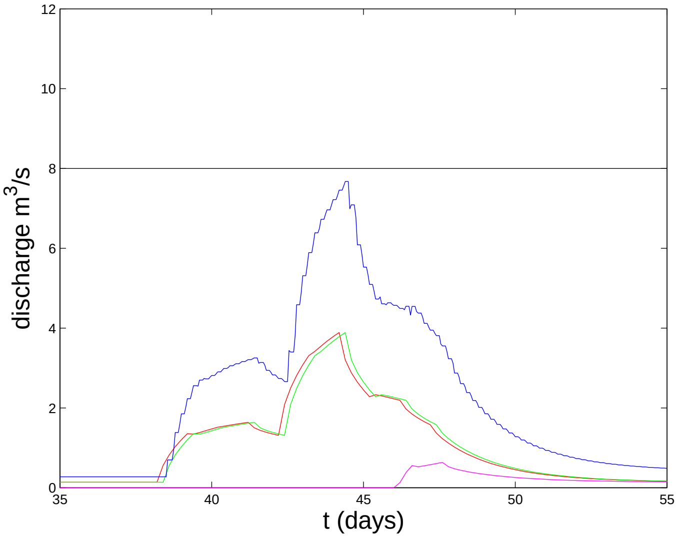

Figure 3.11: Discharge rates with weir height 0.6 m and initial ditch level 0.25 m for one subcatchment (red).

sensitivity analysis can be easily read in Figure 3.12: An increase in ρ_i strongly increases the height of the peak in the discharge curve D_i and flattens the tail. And reversely, if ρ is decreased the discharge curve will get a lower peak and a thicker tail. This agrees with the interpretation of ρ as the parameter measuring the fraction of water fallen on some day that is discharged that same day.

3.5 Recommendations

The discussions above yield the insight that changing the ρ_i parameter of a subcatchment influences the height of the discharge curve but does not influence the respective peak and tail positions in the discharge curve. Since the delay times τ_i are relatively small compared to the widths of the peaks in rainfall and discharge curves, the peaks in the discharge curves D_i all accumulate in the peak of the Regge discharge curve D and the same holds for the tails. This immediately leads to the following recommendation:

In case of intensified peaks in the rainfall due to climate change, the ρ value of a number of subcatchments should be decreased.

The implementation of this recommendation requires some subtle considerations, which we summarize in the following remarks:

Remark a.: Reduction of the ρ value of a subcatchment implies that the drainage of the area should decrease. This could be achieved by closing some ditches or by

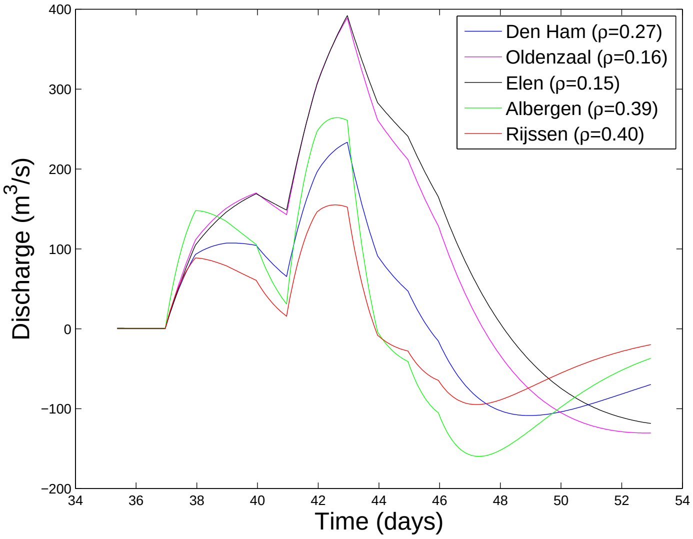

Figure 3.12: Time behavior of the derivative of a typical discharge curve with respect to the parameter ρ.

raising the water level in the drainage ditch network, by increasing the height of the weir, which at the same time increases the water storage capacity of the soil.

Remark b.: It does not matter whether ρ is reduced by a great amount in a relatively small number of subcatchments or if ρ is reduced by a small amount in many subcatchments. The total effect is in both cases nearly the same.

Remark c.: Reducing ρ in some subcatchments reduces the peak height in the Regge discharge curve, but enhances also its tail. So, the optimal choice must follow from a balance between these effects. The total effect of reducing values ρ_i should be such that the peak height in the Regge discharge curve remains under the critical value, dictated by the risk of flooding along the Vecht, and, at the same time, the tail in the Regge discharge curve should remain so low that no dangerous interference with the peak in the Vecht discharge curve occurs. This is a subtle balance. Since the choice of the subcatchments that are most suitable for a change in drainage capacity heavily depends on the local conditions and possibilities, we have not worked out this choice in detail.

Remark d.: The effect of the time delays τ_i is relatively small. If one would like to make use of the fact that the subcatchments differ in this aspect, one could best reduce the ρ parameter in the subcatchments with the largest time delays; the ones furthest away from the discharge point of the Regge into the Vecht. This is because their peaks would arrive latest at the discharge point and thus would interfere most with the peak in the Vecht discharge curve.

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