Optimizing a closed greenhouse

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Abstract

In this project we study the optimization of a closed greenhouse. Typical components of such a greenhouse are aquifers in which surplus heat (in summer) or cold (in winter) is stored. Water tanks are used to control short term variations in the heat and radiation supply due to the daily weather variability. A closed greenhouse may also yield an excess of electricity which is delivered to the public grid. The energy economics is determined by a considerable number of components. The full optimization of such a greenhouse involves the incorporation of market prices of crops and of electricity and gas, and the weather conditions. A formidable problem, as most of these inputs are stochastic variables. We therefore restrict ourselves to minimizing the energy costs given the heat and cold demand for a typical year. Discretizing the model equations on an hourly time basis, we show that this problem has a linear cost function which has to be optimized under linear conditions. Standard linear programming techniques are therefore applicable, guaranteeing our limited optimization problem of a closed greenhouse to be tractable.

Key words: optimization, energy conservation, linear programming, greenhouse, mathematical modelling

1 Introduction

The concept of using greenhouses has a long history. It allows farmers to grow products during seasons in which and at places where the climate conditions are not or far from optimal for the crop under consideration. In Europe this branch of agriculture is very dynamic. The competition is huge, especially as frontiers between the European countries become more and more open. Furthermore, the costs of energy are continuously increasing and governmental regulations to protect the environment of the greenhouses against negative influences become more and more severe. In 1997, within the framework of a project called 'Greenhouse of the future', a number of innovations were introduced in the classic greenhouse concept. These developments led to the idea of GeslotenKasÂő or 'closed greenhouse'. See Fig. 1.1 for a view from the outside and Fig. 1.2 for an inside view of such a closed greenhouse. Since 2003 this new type of greenhouse has been sold and further developed by Innogrow [1]. It is especially suitable for the growth of products that need climatic conditions that

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vary only slightly, and for which CO_2 is an important growth factor. The new concept has economic advantages, especially for products that consume a lot of energy, such as some vegetables, roses, and orchids. The relevant growth factors in most greenhouses depend on temperature, humidity,

Figure 1.1: *Outside view of a Closed Greenhouse.*

and CO_2. By closing the windows of a greenhouse and providing it with an integrated climate and energy system, maximal control is obtained over these growth factors. For example, the CO_2 can be kept at a high level, which favours plant growth in general. Furthermore, the energy use can be kept under control, and thus optimized. An extra advantage of keeping the greenhouse closed is the lower susceptibility to diseases that might invade the greenhouse via the atmosphere. All of these aspects result in a lower energy use, together with an increase in production.

A greenhouse could be considered as a huge sun collector. Unfortunately, the radiation of the sun is not uniformly distributed over the year. In the summer there is an excess and in winter a lack of heat. Apart from the annual cycle, short-term temperature variations take place due to local weather conditions. Similar variations are found for humidity and CO_2. The idea of a closed greenhouse is to smooth these differences out by carefully controlling the local conditions.

1.1 Optimization of a closed greenhouse

The ultimate goal of optimizing the profit of a closed greenhouse includes two aspects:

- Maximizing crop growth and harvest at the right time.
- Minimizing operation costs, i.e., the energy demand to heat/cool the greenhouse.

These two aspects require the application of several submodels:

- A climate model, which determines the heating and cooling requirements of the greenhouse given the weather conditions.
- A crop model, which determines the development of the crop given the light, nutrition, CO_2 and humidity conditions.
- An energy cost model, also referred to as *utility model*, which calculates the costs if a certain energy strategy is followed.
- A market model, which predicts when the crop can be sold for a good price.

It has to be realized that several additional aspects are relevant, which make a complete optimization approach very hard, if not nearly impossible. For example, in practice the weather forecast is available for a few days. Within this time horizon an optimization tool that focuses on the energy expenditure could calculate the best strategy, and when time goes on and the weather forecast is updated, the strategy could be updated, too. However, such a short term strategy should be combined with long term issues such as crop and market developments. Furthermore, a highly restrictive condition, stemming from environmental safeguarding considerations, is that on average per year the total amount of heat/cold pumped into and released from the soil is vanishing: the system may not heat up or cool down the soil layer used for storage. So, this leads to a periodic boundary condition: after a year the situation in the storage devices must be the same as at the start of that year. Such a long term condition may have strong consequences for the short-term energy costs strategy.

Figure 1.2: *Inside view of a Closed Greenhouse.*

1.2 Goal of the project

In view of the considerations mentioned above the full treatment of optimizing the closed greenhouse is a huge task. To keep the project tractable we therefore defined a restricted goal: optimization of the energy costs in a typical year. For example, this implies that the influence of CO_2 is neglected. For temperature and radiation conditions, we take the average values over a long time period in the past. As for the crop development, we assume that a constant temperature and humidity are optimal. Typical heat/cold demands follow from these assumptions over the period of a year.

Similar assumptions hold for the energy prices. In summary, we assume to be given (on an hourly basis):

- the heat/cold demands of a ‘typical’ year, and

- the prices of electricity and gas.

Furthermore, the capacities of all storage, pump, and heat generation devices are known. Given these data, the *purpose of the project* is to: *find a heating/cooling strategy such that the costs are minimized, under the conditions that the reservoirs have the same heat/cold contents before and after the optimization period.*

In principle, the period of optimization must be a year, but for practical reasons we could start with a shorter period. It is expected that the models will yield useful information on:

- the capacities needed to cover the requirements in a typical year, and

- the best strategy for a typical year.

The insight, information, and mathematical tools gained from the model are useful in general, in particular in the following two ways:

- they allow to design new greenhouses based on the ‘closed greenhouse’ concept, and

- the mathematical techniques to be developed may directly be applied in a computer tool for the short-term optimization of the energy costs.

Finally, this report is organized as follows. In §2 we describe the components that constitute the energy sources of the closed greenhouse. In §3 the model presented is shown to be linear in the variables after discretization of the time window over which it is optimized. In §4 interim results are summarized, which are discussed in §5.

2 Energy (Utility) Model

In the present project, we focus on the energy needs of the closed greenhouse system. The purpose is to optimize the efficiency of its energy management, such that the costs are minimized. The system has a complicated network of devices to control the energy fluxes. In Fig. 2.1, a sketch is given of the energy devices in the closed greenhouse.

2.1 The simplified utility model

There are four basic demands: the heat and cold demands together with the demands for light and CO_2 for the crop. Here, as a first step we have decided to neglect the light and CO_2 demands.

Cold and hot water storages

Long term heat excesses in the greenhouse are stored in an underground aquifer, and short term heat excesses in a water reservoir alongside the greenhouse. Similar storage devices are used to store and release cold. The transfer of heat is performed by a heat exchanger.

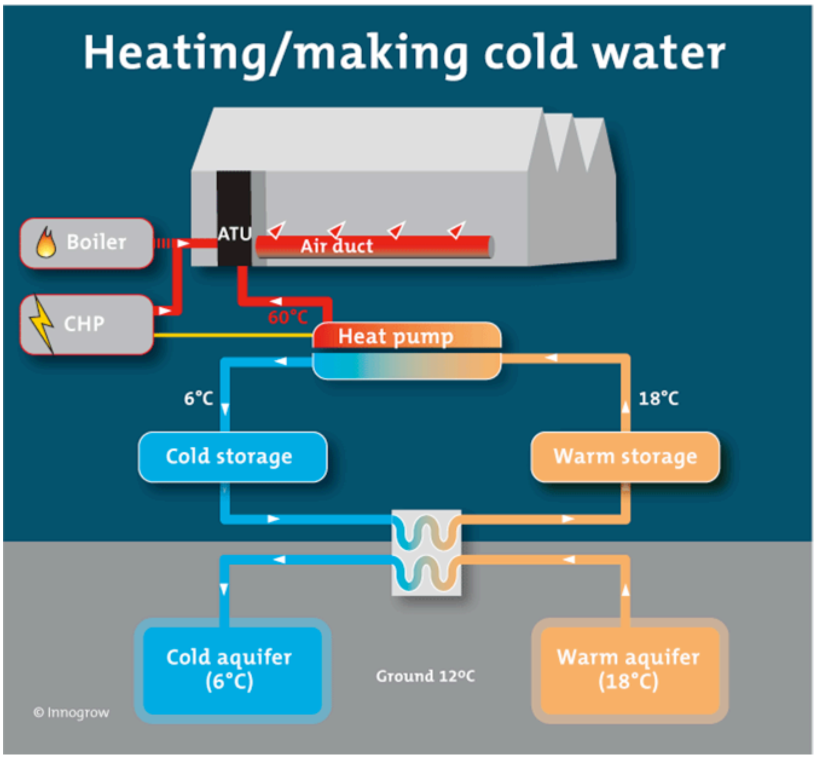

Figure 2.1: *Schematic of the components involved in the energy needs of the Closed Greenhouse concept.*

Heat from electricity

A consequence of our simplified model is that electricity is used only for driving the heat pump (ignoring the requirement for providing light at night). This pump has the property that it can only transfer heat and cold together, not independently. We have then to decide whether to buy electricity from the public grid or to produce it from the "combined heat power" (CHP) unit (also known as cogeneration). It is depicted in Fig. 2.2. The surplus of electricity can be delivered to the public electricity network, but the price of electricity delivered to the network is lower than the price of electricity bought from this network. Different day and night electricity prices must also be taken into account.

Heat from gas

The CHP device consumes gas (G^c). It produces not only electricity but also a heat quantity Q^c proportional to G^c. The system also contains a boiler that consumes gas and produces heat (Q^{bl}). The CO_2 generated by these two devices is not taken into account as the CO_2 demand for the crop is neglected.

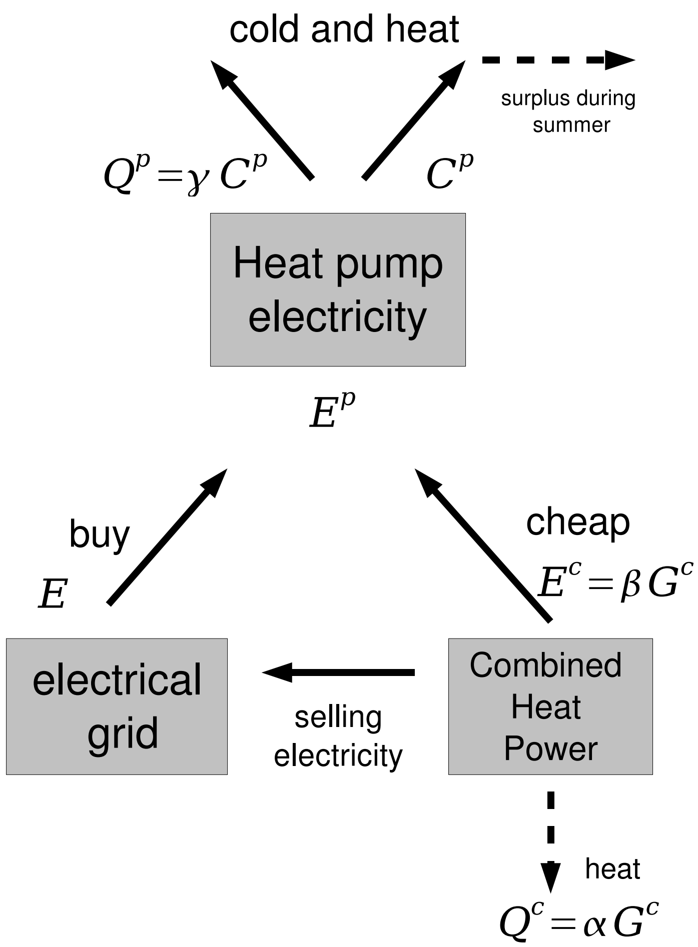

Figure 2.2: *Part of the energy network of the Closed Greenhouse, showing the relations between the heat pump, the combined heat power unit and the external electrical grid.*

2.1.1 Seasonal requirements

Accessing the aquifer is a very expensive initial investment, but later on it is a relatively cheap source of heat and cold, so it should be used as much as possible. What has to be done during the summer is quite clear. We only use the cold aquifer as a cheap source of cooling. Conversely, during winter, the aquifer is always configured as a heat supply. The difficulties arise with intermediate seasons where the aquifer may be switched from hot to cold according to needs. It must be noticed that switching the aquifer has a cost. During the 15 minutes after a switch, the water coming from the aquifer contains sand and cannot be used. Hence gas must be consumed to compensate the heat demand. We should also mention that during summer, we still have a demand for heat as well as cold. In addition, the air conditioning system needs to reheat greenhouse air after dehumidifying it.

2.1.2 Notation

It is clear that the system is quite intricate in view of the many couplings between the components. As for the notation, we denote the required (and prescribed) cooling demand in the greenhouse by $C = C(t)$ and the heating demand by $Q = Q(t)$.

In the system, the heating and cooling devices are uncoupled nearly everywhere, with the exception of the heat pump, which produces heat and cold at the same time. In the following, we denote an energy flux used for cooling by C. The source of such a flux is indicated with a superscript. E.g., C^p, C^a, and C^b are cooling fluxes stemming from the heat pump, the aquifer, and the cooling buffer, respectively. Heating fluxes are represented by Q. E.g., Q^p, Q^a, Q^b, Q^{bl}, Q^c indicate the heating contribution from the heat pump, the aquifer, the heating buffer, the boiler, and the CHP unit, respectively. The total heat flux from and to the aquifer is denoted by q^a, which can be positive (from) and negative (to). Electricity from the grid is denoted by E, it can be either positive (if supplied from grid to the greenhouse) or negative (the other way around). Electricity from the CHP is denoted by E^c. Gas used by the CHP is denoted by G^c and gas used by the boiler is related to the heat flux of the boiler Q^{bl} with efficiency and gas price conversion factors.

There are storage tanks or so-called *buffers* (relatively small compared to the aquifers), for short-term heating and cooling with fluxes C^b (cold temperature storage) and Q^b (warm temperature storage). The energy levels in the cold and hot buffer are H^{bc} and H^{bh}. The energy level in the aquifer is denoted by H^a.

2.2 Energy balances

In the following, we summarize the energy balances in the system. The total cooling demand $C = C(t)$ is supplied by cold fluxes from the heat pump, the aquifer and the cold buffer:

$$C = C^p + C^a + C^b. \tag{2.1}$$

The total heating demand $Q = Q(t)$ is supplied by heat fluxes from the heat pump, aquifer, hot buffer, boiler, and CHP:

$$Q = Q^p + Q^a + Q^b + Q^{bl} + Q^c. \tag{2.2}$$

The aquifer total flux q^a (a negative or positive value) is related to C^a and Q^a by

$$C^a = \max(-q^a, 0), \qquad Q^a = \max(q^a, 0). \tag{2.3}$$

The energies in buffer and aquifer are related to the fluxes according to:

$$\frac{\mathrm{d}H^a}{\mathrm{d}t} = q^a, \tag{2.4}$$

$$\frac{\mathrm{d}H^{bh}}{\mathrm{d}t} = -Q^b - \mu_h H^{bh}, \tag{2.5}$$

$$\frac{\mathrm{d}H^{bc}}{\mathrm{d}t} = -C^b - \mu_c H^{bc}, \tag{2.6}$$

in which we introduced damping coefficients μ_h and μ_c associated with heat or cold losses. The heat pump has the special property that it produces heat and cold at the same time and at a fixed ratio:

$$Q^p = \gamma C^p \qquad \text{with} \qquad \gamma > 1. \tag{2.7}$$

The cooling power plus electrical demand of the heat pump equals its heating power:

$$C^p + E^p = Q^p. \tag{2.8}$$

The electricity used by the heat pump stems from the grid and the CHP (combined heat power):

$$E^p = E + E^c. \tag{2.9}$$

The gas used by the CHP yields both heat production and electrical power at fixed ratios:

$$\alpha G^c = Q^c, \quad \beta G^c = E^c, \tag{2.10}$$

with α and β the proportionality constants. In practice, the efficiency of the CHP is not 100 %, so that in general $\alpha + \beta < 1$. Because of efficiency considerations the boiler either operates at a maximum or is shut off; hence

$$Q^{bl} = 0 \quad \text{or} \quad Q^{bl} = \widetilde{Q}^{bl} \tag{2.11}$$

with $\widetilde{Q}^{bl}$ a fixed value. If the CHP is working this happens at at least 50% capacity:

$$G^c = 0 \quad \text{or} \quad G^c \in [0.5, 1]\,\widetilde{G}^c. \tag{2.12}$$

In summary, the balance equations of the system are

$$C = C^p + C^a + C^b, \tag{2.13a}$$
$$Q = Q^p + Q^a + Q^b + Q^{bl} + Q^c, \tag{2.13b}$$
$$C^a = \max(-q^a, 0), \tag{2.13c}$$
$$Q^a = \max(q^a, 0), \tag{2.13d}$$
$$\frac{\mathrm{d}H^a}{\mathrm{d}t} = q^a, \tag{2.13e}$$
$$\frac{\mathrm{d}H^{bh}}{\mathrm{d}t} = -Q^b - \mu_h H^{bh}, \tag{2.13f}$$
$$\frac{\mathrm{d}H^{bc}}{\mathrm{d}t} = -C^b - \mu_c H^{bc}, \tag{2.13g}$$
$$Q^p = \gamma C^p, \tag{2.13h}$$
$$C^p + E^p = Q^p, \tag{2.13i}$$
$$E^p = E + E^c, \tag{2.13j}$$
$$\alpha G^c = Q^c, \tag{2.13k}$$
$$\beta G^c = E^c, \tag{2.13l}$$

Note that there are 15 variables involved in (2.13): C^p,C^a,C^b,Q^p,Q^a,Q^b, Q^{bl},Q^c,q^a,H^a,H^{bh}, H^{bc},E^p,E^c,E^p. The range limits are denoted as:

$$0 \le H^a \le \widetilde{\mathcal{E}}^a, \qquad 0 \le H^{bh} \le \widetilde{H}^{bh}, \qquad 0 \le H^{bc} \le \widetilde{H}^{bc} \tag{2.14}$$

$$0 \le C^{\cdots} \le \widetilde{C}^{\cdots}, \qquad 0 \le Q^{\cdots} \le \tilde{Q}^{\cdots}, \tag{2.15}$$

where $C^{\cdots}$ and $Q^{\cdots}$ refer to any of the cold and heat fluxes.

2.3 Linear modelling

The aim is to formulate the problem as a linear program (LP), but some elements of the above formulation are impermissible. It should be clear that some of the equations do not allow a strictly linear model to be constructed. For example, (2.13c) and (2.13d) contain the (nonlinear) *max* function. Such a nonlinearity is circumvented by rewriting the constraints as

$$q^a = Q^a - C^a$$

and including a term

$$\lambda(Q^a + C^a) \tag{2.16}$$

in the cost function, where λ is a positive scalar. This ensures that at least one of the variables will be zero at an optimal solution of the LP; it can readily be seen that in the event that both are positive, the cost function can be decreased by reducing both values until the smaller value is zero, without affecting the constraint. This also relies on the fact that LP variables must be nonnegative—something guaranteed by any method, such as the simplex algorithm. The actual solution obtained will obviously depend on a particular choice of λ. We will defer discussion on this value, and its interpretation until §3.

The nonnegativity requirement also affects some of the variables: for example, the cold buffer flux as formulated is a directional variable with its value unconstrained in sign. This can be dealt with simply by decomposing it as

$$C^b = C^{b^+} - C^{b^-}$$

and then using a Boolean variable δ in a pair of constraints:

$$C^{b^+} \leq M\delta \qquad \text{and} \qquad C^{b^-} \leq M(1-\delta),$$

where M stands for a suitably large scalar—larger than the maximum value that can feasibly be taken by the flux. The effect of this is that at most one can be positive, corresponding to the relevant direction of the flux. A similar approach is used for the hot buffer and the grid supply of electricity.

Finally, there are some either/or constraints that also need to be modelled using Booleans. For example, if the CHP is on, it must operate at more than 50% of its maximum capacity. This can be formulated as a pair of constraints:

$$G^c \leq \delta\widetilde{G}^c \qquad \text{and} \qquad G^c \geq 0.5\delta\widetilde{G}^c.$$

The situation is similar for the boiler, except that in this case, if it operates at all, it is at maximum capacity, so that only one constraint is needed:

$$Q^{bl} \leq \delta\widetilde{Q}^{bl}.$$

2.4 Cost function

The cost function (in euros) is given by the following integral over the time window $[0, T]$) considered:

$$K = \int_0^T [f(t', \text{sign}(E))\; E(t') + c_1\, G^c(t') + c_2\, Q^{bl}(t')]\, \mathrm{d}t', \tag{2.17}$$

where the first term in the integrand $f(t, \text{sign}E))\; E(t)$ is the cost of the electricity and t is time. The coefficient $f(t, \text{sign(E)}) > 0$ attains four positive values and is thus boolean in nature. It takes different values for day and night. Further, its sign indicates whether the electricity is taken from or delivered to the electricity grid. To wit:

$$f(t, E) = \begin{cases} \begin{cases} f_1 & \text{day hours} \\ f_2 & \text{night hours} \end{cases} & \text{for} \quad E > 0 \\ \begin{cases} f_3 & \text{day hours} \\ f_4 & \text{night hours} \end{cases} & \text{for} \quad E \leq 0 \end{cases} \tag{2.18}$$

with $f_1 > f_3$ and $f_2 > f_4$.

Further weak constraints are that the cold and warm buffers have no net influx over short periods, while the aquifer must remain in heat balance over a long period T_y, a year, say:

$$\int_0^T C^b(t')\mathrm{d}t' \approx 0, \qquad \int_0^T Q^b(t')\mathrm{d}t' \approx 0, \qquad \text{and} \qquad \int_0^{T_y} q^a(t')\mathrm{d}t' \approx 0. \tag{2.19}$$

Finally, we must remember to add the parameterized term from (2.16), which resurrects the question of the meaning of λ. Given its context, it should be clear that this relates to the cost of extracting heat from (or storing it in) the aquifer. A reasonable approximation could perhaps be determined after some lengthy calculations. Initially it was assumed by Innogrow that this cost is negligible, being dominated by the other sources, so it may be sufficient to try some rough estimates to determine the boundaries within which a particular solution remains optimal.

3 Discretization and linear programming

3.1 Linear programming

The most important insight for finding an optimization procedure for the cost function under the conditions given by the balance equations, is that both the cost function and the conditions are essentially linear in the variables. To see this for the cost function, we simply replace the integral by a Riemann sum taking as grid points hourly intervals. So, if we introduce as variables the hourly values of the 15 variables, the cost function is linear in this function space. Since the balance equations are instantaneous relations between variables, they are inherently linear in the hourly variables, too. Therefore we decided to apply linear programming techniques. The only complicating factor is that some variables are Boolean. Fortunately, standard linear programming techniques such as the 'simplex' method have been extended to incorporate Boolean variables. The idea of linear programming is to find the extremum of the cost function by ascending/descending along the nodes of a multidimensional simplex. Owing to the linearity this can be done in a systematic way using a simple 'greedy' algorithm to move from one node to the next. Although theoretically the computational complexity of this approach is not polynomial, practical experience over 5 decades has shown that the average-case performance is very good, and optimal solutions can be obtained relatively quickly. Adding a moderate number of Booleans degrades the performance a little; if very many Booleans are required then it may take a very long time to find an optimum. However, even in such cases solutions of excellent quality are usually found quickly—the problem is in proving that the quality is good!

3.2 Algebraic system in linear form

Note that by using hourly variables, the total number of variables is very high, especially if the time period is taken to be very long. For example, 15 variables taken on an hourly basis during a year leads to $15 \times 24 \times 365 = 131400$ variables in total. Furthermore, since for linear programming all variables must be nonnegative, we have to write some variables as the difference of two nonnegative variables. This increases the number of variables again. However, present day standard packages for linear programming may deal with huge numbers of variables.

In the end, the entire system of equations is rewritten as

$$C_i^b = C^{b+}_i - C^{b-}_i, \tag{3.1a}$$
$$Q_i^b = Q^{b+}_i - Q^{b-}_i, \tag{3.1b}$$
$$C_i = C_i^p + C_i^a + C^{b+}_i - C^{b-}_i, \tag{3.1c}$$
$$Q_i = Q_i^p + Q_i^a + Q^{b+}_i - Q^{b-}_i + Q_i^{bl} + Q^c{}_i, \tag{3.1d}$$
$$q_i^a = Q_i^a - C_i^a, \tag{3.1e}$$
$$H_i^{bc} = H_{i-1}^{bc} - C^{b+}_{i-1}\,\Delta t + \Delta t\, C^{b-}_i - \mu_c\, H_{i-1}^{bc}\,\Delta t, \tag{3.1f}$$
$$H_i^{bh} = H_{i-1}^{bh} - Q^{b+}_{i-1}\,\Delta t + \Delta t\, Q^{b-}_i - \mu_h\, H_{i-1}^{bh}\,\Delta t, \tag{3.1g}$$
$$Q_i^p = \gamma\, C_i^p, \tag{3.1h}$$
$$C_i^p + E_i^p = Q_i^p, \tag{3.1i}$$
$$E_i^p = E_i + E_i^c, \tag{3.1j}$$
$$E_i = E_i^+ - E_i^-, \tag{3.1k}$$
$$\beta\, G_i^c = E_i^c, \tag{3.1l}$$
$$\alpha\, G_i^c = Q^c{}_i. \tag{3.1m}$$

Here, Δt is the time step, usually one hour. Note that the (hourly) heat and cold demands Q_i and C_i are given. The domains of validity and Boolean variables are

$$-\widetilde{q}^a \le q_i^a \le \widetilde{q}^a, \tag{3.2a}$$
$$0 \le E_i^+ \le M\,\delta_{3i}, \tag{3.2b}$$
$$0 \le E_i^- \le M\,(1-\delta_{3i}), \tag{3.2c}$$
$$0 \le C^{b+}_i \le M\,\delta_{4i}, \tag{3.2d}$$
$$0 \le C^{b-}_i \le M\,(1-\delta_{4i}), \tag{3.2e}$$
$$0 \le Q^{b+}_i \le M\,\delta_{5i}, \tag{3.2f}$$
$$0 \le Q^{b-}_i \le M\,(1-\delta_{5i}), \tag{3.2g}$$
$$Q_i^{bl} \le \delta_{1i}\,\widetilde{Q}^{bl}, \tag{3.2h}$$
$$G_i^c \le \delta_{2i}\,\widetilde{G}^c, \tag{3.2i}$$
$$G_i^c \ge 0.5\,\delta_{2i}\,\widetilde{G}^c, \tag{3.2j}$$

with $\delta_{1i} = 0, \delta_{2i} = 1$, $\delta_{3i} = \delta_{4i} = \delta_{5i} = 1$ if $E_i, C^{b+}_i, Q^{b+}_i > 0$ and with $\delta_{1i} = 1, \delta_{2i} = 0$, $\delta_{3i} = \delta_{4i} = \delta_{5i} = 0$ if $E_i, C^{b+}_i, Q^{b+}_i = 0$. M is a very large integer.

The cost function (2.17) is discretized in time, over intervals Δt over a period T such that $N\,\Delta t = T$. It then becomes

$$K = K(\mathbf{Q}) = \sum_{i=1}^{N} P_i^+ E_i^+ + P_i^- E_i^- + c_1 G_i^c + c_2 Q_i^{bl} + c_3 (Q_i^a + C_i^a) \tag{3.3}$$

with $P_i^+ = f_1$ or f_2, and $P_i^- = -f_3$ or $-f_4$ for day and night prices of electricity intake or supply from the net. The cost function depends on all variables $\mathbf{Q}$ in the desired period.

The values c_1, c_2 represent the actual costs of gas for the CHP and boiler respectively, while c_3 is what we earlier called λ —the variable cost of using the aquifer.

In summary, the procedure is to minimize cost function (3.3) of the system (3.1) with booleans (3.2) with linear programming (LP).

3.2.1 Possible refinements

The above formulation assumes that we can treat periods (of whatever length) independently, which is probably not entirely true. A few extra constraints came to light in subsequent discussions: for instance, the aquifer cannot work at full capacity after being switched from the cooling to the heating mode or vice versa. A reasonable estimate is that the maximum capacity in such cases would be 75% of its normal value. This necessitates inter-period constraints. For example, in period i we have a new Boolean variable δ_{6i} which is 1 (resp. 0) if the aquifer is in heating (resp. cooling) mode in period i, and constraints

$$Q_i^a \le \widetilde{q}^a\,\delta_{6i} \quad C_i^a \le \widetilde{q}^a\,(1 - \delta_{6i}).$$

This ensures that at most one of Q^a, C^a is positive, and both are bounded from above. Then we have to consider whether the aquifer was in the heating mode in period $i-1$ or not. If it was, we can use the full capacity, otherwise only 75%. The constraint

$$Q_i^a - 0.25\,\delta_{6(i-1)}\,\widetilde{q}^a \le 0.75\,\widetilde{q}^a$$

will model this situation. There is an analogous constraint for the cooling mode.

4 Optimization experience

For the parameters we used the following values [2]

$$\begin{aligned}(\bar{C}, \bar{Q}) = {} & (70, 43)\,\mathrm{W/m^2}, \quad (C^{obs}{}_{max}, Q^{obs}{}_{max}) = (629, 124)\,\mathrm{W/m^2},\\ \alpha = {} & 0.501, \quad \beta = 0.42,\\ COP = {} & 4.2, \quad \delta = 0.97,\\ c_1 = {} & 0.0262\,\mathrm{Euro/kWh},\\ c_2 = {} & 0.029\,\mathrm{Euro/kWh},\\ (f_1, f_2, f_3, f_4) = {} & (0.109, 0.059, 0.085, 0.042)\,\mathrm{Euro/kWh}.\end{aligned} \tag{4.1}$$

The optimization program was run with the above values and c_3 running over the values 0.001, 0.01 and 0.1. An initial attempt was made to verify the formulation using the simple LP package LINDO—a "student" version of which is freely downloadable [3]. This version proved sufficient to deal with a day with a known demand profile divided into 3 periods of 8, 10 and 6 hours. With

c_3 = 0.001, the LP was poorly scaled and gave rise to numerical problems, but eventually a solution was obtained. Although the actual numerical values changed a little over the range of values used for c_3, it was encouraging that the overall structure of the different solutions (i.e., which sources were used or ignored) tended to remain fairly stable, and agreed in broad terms with what would have been expected. Nonetheless, it suggested that there is a need for a more accurate estimate of the value of c_3 in order to obtain a better approximation of the optimal solution.

Later it was possible to use another software package—CPLEX, which implements linear and integer programming techniques in a significantly more sophisticated way [4]. For example, it can take care of nonnegative variables implicitly, without the need for the 'tricks' used above, thus reducing the number of variables. It is also considerably faster than LINDO. Hence, it was possible to discretize to hourly periods, and to optimize the model over a time horizon of a year. The results were satisfactory, but gave rise to several further problems that suggested the modelling needs some fine-tuning. Although the costs were of the right order of magnitude, they suggested values that were above those currently incurred without optimization! Also in some cases the LP became infeasible after several periods had elapsed. These suggest that the existence of unsuspected inter-period constraints needs to be examined further, and the accuracy of all parameter values needs to be considered again.

5 Conclusions

We conclude that the greenhouse energy optimization problem can be formulated as a linear programming problem including a number of Boolean variables. To find the solution standard techniques can be applied. The computation times strongly depend on the time period over which is optimized. In principle it must be possible to take a full year as optimization period. This allows the application of the boundary conditions that are in force, namely that the net influx in the aquifers averaged over a year must vanish. It would also make it possible to find the optimal strategy for a year with "typical" weather conditions, i.e., averaged over a long period. These insights in turn allow the designers to use "typical" dimensions for the capacities of all components, when they are designing a new greenhouse. Furthermore, since the optimization is very fast for short periods, this project makes clear that optimization of the energy costs in a closed greenhouse can easily be performed using standard software. However, further work needs to be done in terms of understanding and modelling the inter-period connections, and the effect of other assumptions made in the initial formulation.

References

[1] See the website http://www.innogrow.nl/

[2] Lou Ramaekers, Innogrow, personal communication (2007).

[3] See the website http://www.lindo.com/

[4] See the website http://www.ilog.com/products/cplex/