Modeling a heart pump

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Abstract

In patients with acute heart failure, the heart can be assisted by the insertion of a mechanical device which takes over part of the heart's work load by pumping blood from the left ventricle, one of the heart chambers, into the aorta. In this project, we formulate a model that describes the effect of such a device on the cardiovascular dynamics. We show that data for the pressure–volume relationship within a heart chamber that have been obtained experimentally can be reproduced quite accurately by our model. Moreover, such experimental data can help in calibrating unknown parameters that specify the characteristics of the pump. A key parameter turned out to be the extra friction that is encountered by the blood flowing through the heart pump.

Key words: cardiovascular system, rotary heart pump.

1 Introduction

Nowadays it is possible to use mechanical devices to assist the pumping action of the heart in patients with cardiovascular problems. A particular class of these, left ventricular assist devices, move blood from the left ventricle (one of the heart chambers) into the aorta, to take over part of the heart's work load. Developments in the emerging field of cardiovascular medicine have led to the availability of a wide range of instruments, from temporary assist devices to devices for long-term support and from left or right ventricular assist devices to biventricular assist devices and even total artificial hearts. This report will focus on the effects of a left ventricular assist device called the Impella, manufactured by Abiomed Europe (GmbH, Aachen, Germany). Two versions are currently being used by the Academic Medical Centre (AMC) in Amsterdam: the Impella 2.5 and the Impella 5.0, which are able to produce a flow of 2.5 and 5.0 liters per minute, respectively. The problem considered here has been formulated by cardiologists and cardiothoracic surgeons from the AMC.

The insertion of a mechanical device in the cardiovascular system obviously influences the dynamics of the blood flow through the arterial system. The contraction and relaxation of the heart muscles in the heart chamber causes two valves, called the mitral and aortic valve, to open and close due to pressure differences. When modeling the cardiac cycle, one can distinguish four phases.

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1. Isovolumic contraction phase, when the mitral and aortic valves are closed. Pressure is being built up in this phase, until the left ventricular pressure rises sufficiently above the aortic pressure to open the aortic valve.

2. Ejection phase, when the mitral valve is closed and the aortic valve is open. Blood flows out of the chamber into the aorta.

3. Isovolumic relaxation phase, when the mitral and aortic valves are closed. Pressure in the chamber decreases until it is so low that the mitral valve opens.

4. Filling phase, when the mitral valve is open and the aortic valve is closed. Blood flows into the chamber, and the cycle then repeats itself.

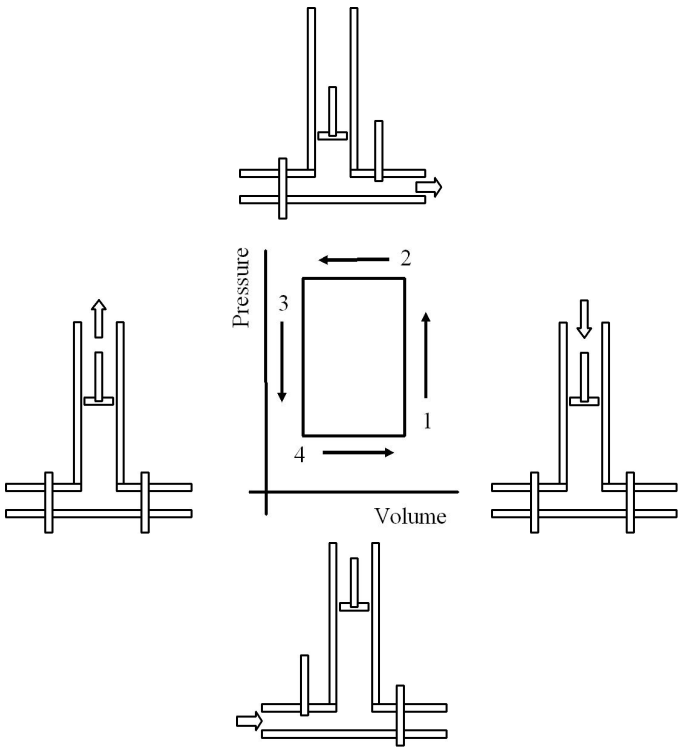


Table 1: The cardiac cycle.

The first two phases are known as diastole and the last two phases are known as systole.

In patients, the influence of a mechanical device which assists the heart during this process is difficult to quantify, since only a limited number of direct measurements can be performed on the cardiovascular system with and without the blood pump. Typically, only the pressure and volume of the blood in a heart chamber can be measured. The questions whether the pump really takes over a substantial part of the task of the heart and whether it reduces the amount of work the heart has to perform cannot be answered directly from such measurements. The amount of flow produced by the heart, the cardiac output, is 5 to 7 liters per minute in healthy people. It can be indirectly determined

by means of a method called thermodilution, which represents the gold standard in clinical practice. However, it is uncertain if this method is reliable in the presence of a continuous flow pump, since the normal physiology of the heart is altered, changing the premisses on which the method is based. Moreover, it is not possible to determine directly the individual contributions of the heart and the pump to their combined cardiac output, since detailed information about certain key parameters in the system is unavailable. Information about the cardiac output and the contribution of the pump are important to assess, and some form of mathematical modeling is therefore indispensable to obtain information about these quantities.

2 Problem Formulation

In this project, the study group was asked to develop a mathematical model for the influence of the Impella on the cardiovascular dynamics. More specifically, the group was asked to focus on the estimation of the blood flow through the pump during the cardiac cycle, since this is an important monitoring variable that can be used to establish how much the work load of the heart can be relieved. To validate the model and to make it possible to calibrate unknown parameters, a PV (Pressure–Volume) loop of a patient with and without the blood pump and an exact specification of the pump had been made available.

The distinguishing characteristic for this problem is the specific placement of the pump across the aortic valve. In order to obtain a realistic model, it is important that the pumping action of the heart itself, the dynamics of the pump, and the rest of the body, i.e. the arterial system, are modeled with sufficient accuracy. Moreover, the problem formulation suggests that the model should not be too complicated or detailed, since we would like to be able to explain the change in qualitative behaviour of the system in direct terms.

The structure of the paper is as follows. In the next section, the heart and the arterial system are modeled, using an analogy with electrical circuits. In section 4, some details concerning the operation of the pump are given. Section 5 combines all these elements in a full cardiovascular model and section 6 provides numerical results. We end by formulating our conclusions and recommendations for further research in the last section.

3 Modeling the Heart and Arterial Systems

The cardiovascular system can be described in terms of quite complex fluid dynamics. Different models have been developed to investigate it and various tools, ranging from rather simple to very sophisticated numerical techniques, have been employed; see for example [9, 11, 16] and references therein for an overview of computational methods in cardiovascular fluid dynamics.

A computationally cheap option to obtain information about the overall behaviour of the cardiovascular system is provided by so-called lumped parameter models [3, 13, 14, 15]. In these models, critical parameters are defined by taking averages over many different subsystems without distinguishing these subsystems themselves in too much detail. Such models proved to be very useful as a starting point for the investigation of arterial blood pressure and blood flow.

There is a close correspondence between the cardiovascular system and electrical circuits, which we intend to exploit here. A description of this correspondence will therefore be given in the following subsections.

Cardiovascular	Electrical
blood volume	electrical charge
flow rate (F)	current (I)
pressure (P)	potential (V)

Table 2: The analogy between the cardiovascular and electrical systems.

3.1 Mapping Cardiovascular Elements to Electrical Elements

In the analogy that we will use, electrical charge represents blood volume, while potential (difference) and currents correspond to pressure (difference) and flow rates. A particular vessel, or group of vessels, can be described by an appropriate combination of resistors, capacitors, and inductors. Blood vessels' resistance, depending on the blood viscosity and the vessel diameter, is modeled by resistors. The ability to accumulate and release blood due to elastic deformations, the so-called vessel compliance, is modeled by capacitors. The blood inertia is introduced using coils, and finally heart valves (forcing unidirectional flow) are modeled by diodes. In the following, we will explain in more detail how the different properties and parts of the cardiovascular system can be modeled by electrical components.

Vessel Resistance and Electrical Resistance

Blood flowing from wider arteries into smaller arterioles encounters a certain resistance. This resistance can be modeled as follows. Consider an ideal segment of a cylindrical vessel. The pressure difference between its two ends and the flow through the vessel depend on each other. Although this dependence will in general be nonlinear, for a laminar flow (which is the type of flow we are interested in) it can be accurately approximated by a linear relation. If we indicate by R_c the proportionality constant between the pressure difference P and the flow F then we can write

$$R_c = \frac{P}{F}. \tag{1}$$

Similarly, a resistor is an electronic component that resists an electric current by producing a potential difference between its end points. In accordance with Ohm's law, the electrical resistance R_e is equal to the potential difference V across the resistor divided by the current I through the resistor:

$$R_e = \frac{V}{I}. \tag{2}$$

Vessel Compliance and Capacitance

The walls of blood vessels are surrounded by muscles that can change the volume and pressure in the vessel. Consider the blood flow into such an elastic (compliant) vessel. We denote the flow into the vessel by F_i and the flow out of the vessel by F_o. Then the difference $F = F_i - F_o$ which corresponds to the rate of change of blood volume in the vessel is related to a change of pressure P inside the vessel. Assuming a linear relation, we have that

$$F = C_c \frac{dP}{dt}, \tag{3}$$

where C_c is a constant related to the compliance of the vessel.

The analogy with a capacitor is immediate. A capacitor is an electrical device that can store energy between a pair of closely-spaced conductors, so-called 'plates'. When a potential difference is applied to the capacitor, electrical charges of equal magnitude but opposite polarity build up on each plate. This process causes an electrical field to develop between the plates of the capacitor which gives rise to a growing potential difference across the plates. This potential difference V is directly proportional to the amount of separated charge Q (i.e. $Q = C_e V$). Since the current I through the capacitor is the rate at which the charge Q is forced onto the capacitor (i.e. $I = dQ/dt$), this can be expressed mathematically as:

$$I = C_e \frac{dV}{dt}, \tag{4}$$

where the constant C_e is the electrical capacitance of the capacitor.

Blood Inertia and Inductance

Since blood is inert, it follows that when a pressure difference is applied between the two ends of a long vessel that is filled with blood, the mass of the blood resists the tendency to move due to the pressure difference. Once more assuming a linear relation between the change of the blood flow (dF/dt) and the pressure difference P we can write

$$P = L_c \frac{dF}{dt}. \tag{5}$$

Note that this is the hydraulic equivalent of Newton's law, which relates forces to acceleration.

The inertia of blood can be modeled by a coil (also known as an 'inductor'), since the current in a coil cannot change instantaneously. This effect causes the relationship between the potential difference V across a coil with inductance L_e and the current I passing through it, which can be modeled by the differential equation:

$$V = L_e \frac{dI}{dt}. \tag{6}$$

Valves and Diodes

An ideal valve forces the blood to flow in only one direction. More specifically, it always stops the flow in one direction while it allows the blood to flow in the other direction, opposing only a small resistance (R_c) to the flow, as soon as the pressure difference is higher than a certain critical pressure P^* which is often taken to be zero. For this reason it is common use to model the action of a valve as follows:

$$F = \begin{cases} 0 & \text{if } P < P^* \\ P/R_c & \text{if } P \geq P^*. \end{cases} \tag{7}$$

The electrical analogue of a valve is a diode. In electronic circuits, a diode is a component that allows an electric current I to flow in one direction, but blocks it in the opposite direction. There are different models in the literature for diodes; we will use the idealized relationship corresponding to (7):

$$I = \begin{cases} 0 & \text{if } V < V^* \\ V/R_e & \text{if } V \geq V^*. \end{cases} \tag{8}$$

3.2 Other Relationships

One advantage of modeling the cardiovascular system by an electrical circuit is that Kirchhoff's laws for currents and potential differences can be applied:

- The sum of currents entering any junction is equal to the sum of currents leaving that junction (conservation of blood mass).
- The sum of all the voltages around a loop is equal to zero (pressure is a potential difference).

All these elements, or their nonlinear extensions, are used in different forms in the models for the heart and its environment, the arterial system. We summarize all the relationships in table 3.

In the following subsection, we discuss the relatively simple models for the cardiovascular system that are known as Windkessel models.

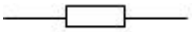
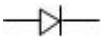
$P = FR_c$	vessel resistance	elec. resistance 	$V = IR_e$
$C_c \frac{dP}{dt} = F$	vessel compliance	elec. capacitance 	$C_e \frac{dV}{dt} = I$
$L_c \frac{dF}{dt} = P$	blood inertia	magnetic inductance 	$L_e \frac{dI}{dt} = V$
$F = \begin{cases} 0 & \text{if } P < 0 \\ P/R_c & \text{if } P \geq 0 \end{cases}$	valve	diode 	$I = \begin{cases} 0 & \text{if } V < 0 \\ V/R_e & \text{if } V \geq 0 \end{cases}$

Table 3: Analogy between electrical and cardiovascular behaviour.

3.3 Description of the Windkessel Model and its Use

The Windkessel model consists of ordinary differential equations that relate the dynamics of aortic pressure and blood flow to various parameters such as arterial compliance, resistance to blood flow and the inertia of blood. We discuss three forms of the model. In the different forms, the complexity of the model is increased by introducing extra components, each representing a characteristic of the cardiovascular system. Thus, closed-form solutions for the aortic pressure and the flow rate become increasingly difficult to obtain.

2-Module Windkessel Model

The first Windkessel model was put forward by Stephen Hales in 1733 [7]. He assumed that the arteries operate like a chamber in an old-fashioned hand-pumped fire engine (in German *Windkessel* pump) which smoothes the water pulses into a continuous flow. By conducting blood pressure experiments on various animals he was able to perform the first direct measurement of arterial blood pressure.

Hale's analogy between the cardiovascular system and the water pump was enhanced by the German physiologist Otto Frank [5]. His 2-module Windkessel model has since been applied in studies including chick embryos [17] and rats [8]. Figure 1 shows the 2-module Windkessel model

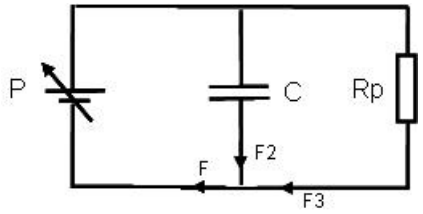


Figure 1: The 2-Module Windkessel model.

consisting of an electrical circuit with a capacitor C corresponding to the arterial compliance and a resistor R corresponding to the resistance to blood as it passes from the aorta to the narrower arterioles. This is referred to as the peripheral resistance. As explained in more detail in section 3.1, P and F represent the aortic pressure and the blood flow rate in the aorta, respectively, and both are functions of time, t. A differential equation in terms of P and F can be obtained using the equations given in section 3.2. These equations lead to

$$F = F_2 + F_3, \tag{9}$$

$$P = F_3 R, \tag{10}$$

$$C\frac{dP}{dt} = F_2. \tag{11}$$

Using Kirchhoff's law for currents, we can eliminate the currents F_2 and F_3 from this equation, leading to:

$$F = \frac{P}{R} + C\frac{dP}{dt}.$$

This equation can be solved if we consider just the diastole period of the heartbeat in which the heart muscles relax, because during this period the left ventricle is expanding and $F = 0$. We then find

$$P = P(t_d)e^{-\frac{(t-t_d)}{RC}}. \tag{12}$$

Here it has been assumed that $P(t_d)$ is the blood pressure in the aorta at the starting time t_d of the diastole.

3-Module Windkessel Model

An extension of the 2-Module Windkessel model, the 3-Module Windkessel model, was formulated by the Swiss physiologist Ph. Broemser together with O. Franke and it was published in an article in

1930 [2]. This model, which is also known as the Broemser model, introduces an extra resistor R_a which represents the resistance encountered by blood as it enters the aortic valve. The corresponding electrical circuit is shown in figure 2.

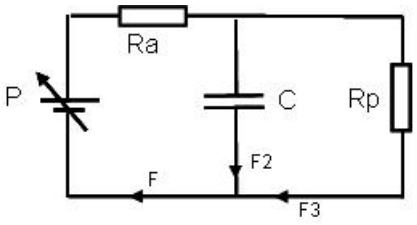


Figure 2: The 3-Module Windkessel model for the systemic circulation.

Adopting the same approach as before we obtain

$$(1 + \frac{R_a}{R_p})F + R_a C_c \frac{dF}{dt} = \frac{P}{R_p} + C_c \frac{dP}{dt} \tag{13}$$

and during diastole, when F and its time derivative are zero, we may again simplify (13). This leads to the same expression for the aortic pressure during diastole as in the 2-Module Windkessel model.

4-Module Windkessel Model

The 4-Module version of the model was developed for the study of the systematic circulation in chick embryos [17] and pulmonary circulation (i.e. the circulation relating to the lungs) in cats [10] and dogs [6]. This model extends the 3-Module version by adding a coil L_c to represent the inertia of the blood, see figure 3. The approach used previously and some tedious but trivial algebra then

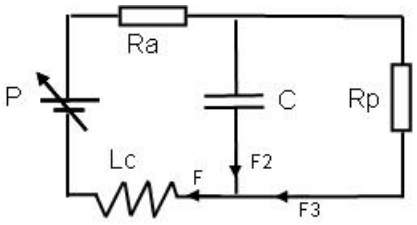


Figure 3: The 4-Module Windkessel model.

results in the following equation:

$$(1 + \frac{R_a}{R_p})F + (R_a C_c + \frac{L_c}{R_p})\frac{dF}{dt} + L_c C_c \frac{d^2F}{dt^2} = \frac{P}{R_p} + C_c \frac{dP}{dt}.$$

In section 5, the elementary Windkessel models will be expanded to include the pumping action of both the heart and the mechanical pump. But first we will give a description of the pump itself and its most important operating characteristics.

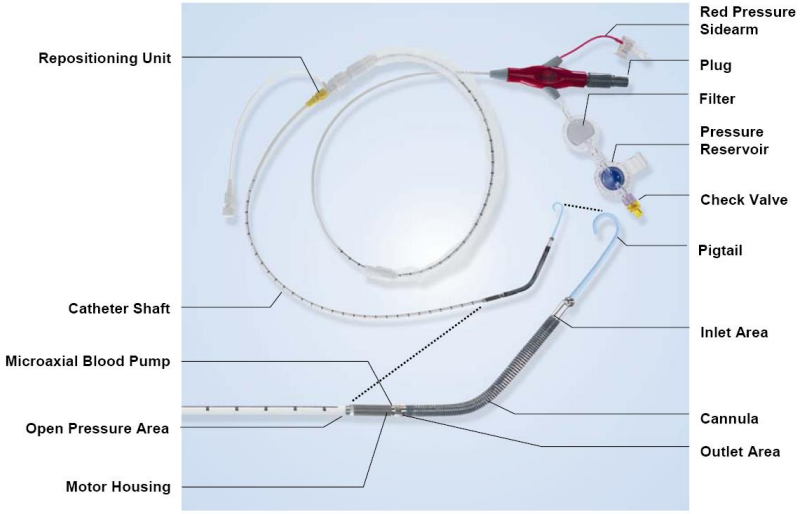


Figure 4: The Impella 2.5 LP device.

4 Description and Specification of the Pump

In this section, we give some specifications which will turn out to be relevant for the mathematical model of the blood pump. Part of the information about the heart pump was obtained from the instruction manual [1] and some was provided by Krischan Sjauw (AMC).

As described earlier, the Impella Recover LP 2.5 is a catheter mounted micro-axial rotary blood-pump, designed for short-term mechanical circulatory support. It is positioned across the aortic valve into the left ventricle, with its inlet in the left ventricle and its outlet in the aorta; see figure 5. The driving console of the pump allows management of the pump speed by 9 gradations and it displays the pressure difference between inflow and outflow, which gives an indication of the pump's position. Expelling blood from the left ventricle into the ascending aorta, the Impella is able to provide a flow of up to 2.5 litres per minute at its maximal rotation speed of 51000 rpm. To prevent aspirated blood from entering the motor, a purge fluid is delivered through the catheter to the motor housing by an infusion pump (see figure 6). Table 4 gives some further specifications.

The flow created by the pump depends mainly on the pressure difference between the outlet in the aorta and the inlet in the ventricle, and on the speed of the rotor. The flow decreases if the pressure difference increases or the rotor speed decreases. Figure 7 shows these dependencies, obtained experimentally, between the flow through the pump and the pressure difference for different pump speeds varying from the maximum possible 51000 rpm to 25000 rpm.

To describe the action of the rotary pump, we need to expand the Windkessel models discussed above to model the valves and the heart chamber, and the dynamics which lead to the cardiac cycle. An example of this is given in [4], where the left ventricle is described as a time varying capacitor with an elasticity function $E(t)$ for the heart, thereby providing a model for the heart capacitance which is time-varying. This time-varying function is calibrated to the end-systolic pressure and volume values and the end-diastolic pressure and volume values.

The pump itself is modeled as a bypass around the diode that represents the aortic valve with

Parameter	Value
Speed range	0 to 51000 rpm
Flow-Maximum	2.3±0.3 l/min
Catheter diameter	max. 4.2 mm (nom. 4.0 mm)
Length of invasive portion (w/o catheter)	130±3 mm
Voltage	max. 18 V
Power consumption	less than 0.99 A
Maximum duration of use	5 days

Table 4: Pump parameters.

resistance and inertia at both ends of the pump. We remark that this means that even when the aortic valve is closed, the pump can still pump blood from the left ventricle into the aorta and there may even occur some backflow: blood flowing back from the aorta into the left ventricle due to adverse pressure differences.

The rotator speed in the pump was assumed to be constant, since this was the explicit design objective of the pump considered here.

5 The Full Model

Our model for the environment of the heart and blood pump is based on a paper by Ursino [12], in which a nonlinear lumped parameter model of the cardiovascular system is proposed. The first order differential equations in this model describe pressures, volumes, and flows in the lumped subsystems.

These subsystems are the pulmonary arteries, pulmonary peripheral circulation, pulmonary veins, systemic arteries, peripheral systemic circulation, extrasplanchnic venous circulation, the left and right atrium of the heart, and the left and right ventricle of the heart. This means a distinction is made between veins, which carry blood to the heart, and arteries, which take blood from the heart to the organs, and between the pulmonary system, which corresponds to the lungs, the splanchnic system, which corresponds to abdominal internal organs, the peripheral system, corresponding to the outer part of the body, and the extrasplanchnic system, which corresponds to other organs.

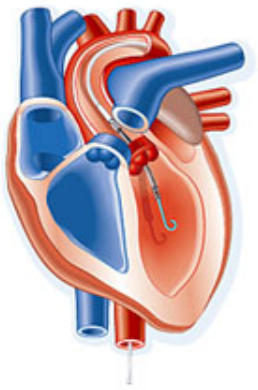


Figure 5: Placement of the pump between the aortic valves.

The Ursino model offers the possibility of parameter adjustments based on physical specifications for individual patients, whereas in a Windkessel model more parameters and state variables are modeled implicitly, and are therefore not adjustable in a straightforward way. Moreover, in the expanded model, blood flow in or out of the right and left ventricles is only possible if the corresponding valves are open. A valve opens one way, i.e. it is opened or closed, depending on the sign of the pressure difference over the valve.

In the model, a distinction is made between the capacitance, inertia, and other characteristics in the different components of the system. For example: the capacitance and resistance for blood flow obviously depend on the width of the arteries and veins, which may vary considerably within the human body. Different compartments of the system are denoted by different subscripts to make the model equations easier to read, see table 5. The electrical circuit corresponding to the model is given in figure 8. Notice that in this figure the time-varying pressures generated by the heart muscles in the left and right ventricle are indicated by capacitors with arrows through them, while we have also used the standard symbol for electrical earth to indicate a point of zero voltage, which corresponds to a reference pressure based on which all other pressure differences are stated.

The model for pressures, volumes and flows then becomes as follows. Conservation of mass and balance of forces in the different compartments lead to

$$\begin{aligned} \frac{dP_{pa}}{dt} &= \frac{1}{C_{pa}}(F_{o,r} - F_{pa}) \\ \frac{dF_{pa}}{dt} &= \frac{1}{L_{pa}}(P_{pa} - P_{pp} - R_{pa}F_{pa}) \end{aligned} \tag{14}$$

$$\begin{aligned} \frac{dP_{pp}}{dt} &= \frac{1}{C_{pp}}(F_{pa} - \frac{P_{pp} - P_{pv}}{R_{pp}}) \\ \frac{dP_{pv}}{dt} &= \frac{1}{C_{pv}}(\frac{P_{pp} - P_{pv}}{R_{pp}} - \frac{P_{pv} - P_{la}}{R_{pv}}) \end{aligned} \tag{15}$$

for the pulmonary arteries in the upper cycle in figure 8, while it leads to

$$\begin{aligned} \frac{dP_{sa}}{dt} &= \frac{1}{C_{sa}}(F_{o,l} - F_{sa}) \\ \frac{dF_{sa}}{dt} &= \frac{1}{L_{sa}}(P_{sa} - P_{sp} - R_{sa}F_{sa}) \\ \frac{dP_{sp}}{dt} &= \frac{1}{C_{sp} + C_{ep}}(F_{sa} - \frac{P_{sp} - P_{sv}}{R_{sp}} - \frac{P_{sp} - P_{ev}}{R_{ep}}) \\ \frac{dP_{ev}}{dt} &= \frac{1}{C_{ev}}(\frac{P_{sp} - P_{ev}}{R_{ep}} - \frac{P_{ev} - P_{ra}}{R_{ev}}) \end{aligned} \tag{16}$$

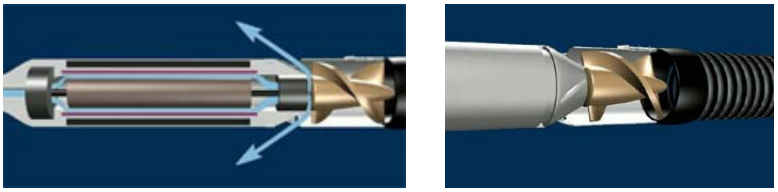


Figure 6: Left: Purge fluid preventing blood from entering motor housing. Right: Rotor positioned above the motor housing.

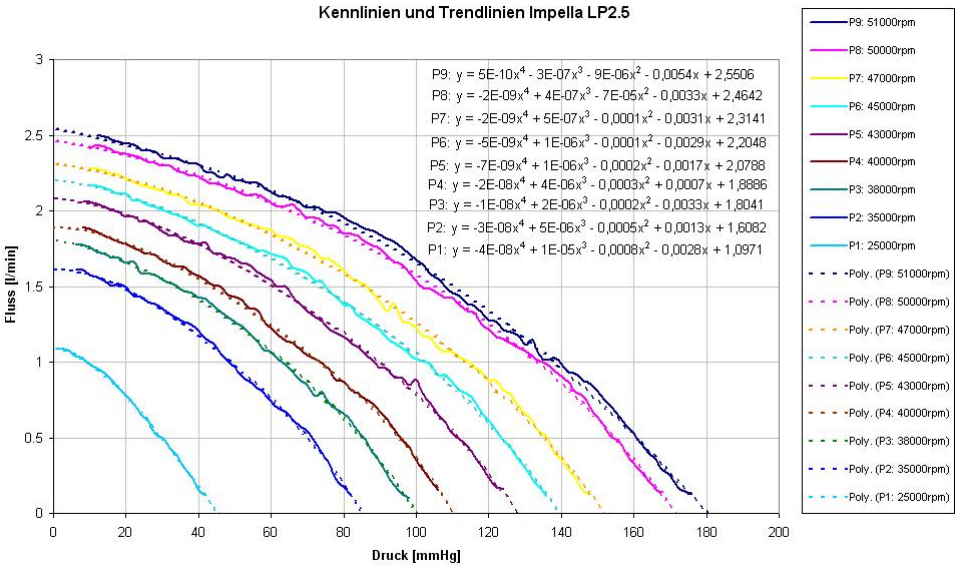


Figure 7: Dependencies between the flow through the Impella LP 2.5 pump and the pressure differences for different speeds of the motor.

for the lower cycle in that figure. Finally, for the left and right atrium

$$\begin{aligned} \frac{dP_{la}}{dt} &= \frac{1}{C_{la}}\left(\frac{P_{pv}-P_{la}}{R_{pv}} - F_{i,l}\right) \\ \frac{dP_{ra}}{dt} &= \frac{1}{C_{ra}}\left(\frac{P_{sv}-P_{ra}}{R_{sv}} + \frac{P_{ev}-P_{ra}}{R_{ep}} - F_{i,r}\right). \end{aligned} \tag{17}$$

Here, $F_{i,l}$ and $F_{o,l}$ are the flow into and out of the left ventricle (in ml/s), and $F_{i,r}$ and $F_{o,r}$ are the flow into and out of the right ventricle. Assuming a known and constant total blood volume V_0 we can express the last remaining pressure, the splanchnic venous pressure P_{sv}, in terms of all the other

pa	pulmonary arteries	pp	pulmonary peripheral
pv	pulmonary veins	sa	systemic arteries
sp	systemic peripheral	ev	extrasplanchnic venous
sv	splanchnic venous	ep	extrasplanchnic peripheral
ra	right atrium	la	left atrium
rv	right ventricle	lv	left ventricle
i	in	o	out
l	left	r	right

Table 5: Subscripts of the variables.

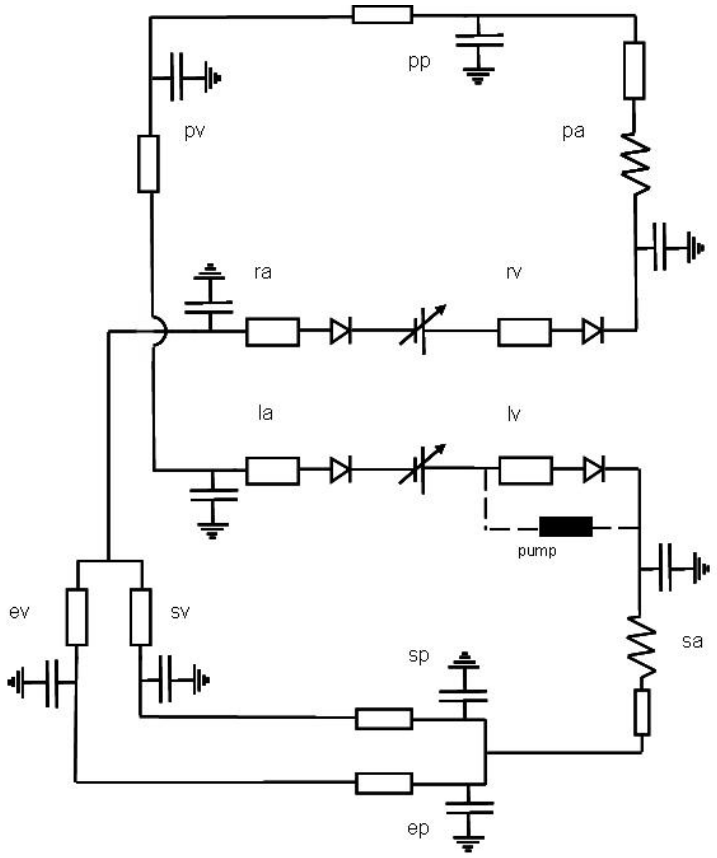


Figure 8: The full model.

pressures:

$$P_{sv} = \frac{1}{C_{sv}}[V_0 - C_{sa}P_{sa} - (C_{sp} + C_{ep})P_{sp} - C_{ev}P_{ev} - C_{ra}P_{ra} - V_{rv} \\ -C_{pa}P_{pa} - C_{pp}P_{pp} - C_{pv}P_{cp} - C_{la}P_{la} - V_{lv} - V_u]. \quad (18)$$

The left and right ventricles are modeled using state variables that represent volumes instead of

pressures and the flows through the valves:

$$\begin{aligned}
\frac{dV_{rv}}{dt} &= F_{i,r} - F_{o,r} \\
F_{i,r} &= \begin{cases} 0 & \text{if } P_{ra} \le P_{rv} \\ \frac{P_{ra}-P_{rv}}{R_{ra}} & \text{if } P_{ra} > P_{rv} \end{cases} \\
F_{o,r} &= \begin{cases} 0 & \text{if } P_{max,rv} \le P_{pa} \\ \frac{P_{max,rv}-P_{pa}}{R_{rv}} & \text{if } P_{max,rv} > P_{pa} \end{cases} \\
\frac{dV_{lv}}{dt} &= F_{i,l} - F_{o,l} \\
F_{i,l} &= \begin{cases} 0 & \text{if } P_{la} \le P_{lv} \\ \frac{P_{la}-P_{lv}}{R_{la}} & \text{if } P_{la} > P_{lv} \end{cases} \\
F_{o,l} &= \begin{cases} 0 & \text{if } P_{max,lv} \le P_{sa} \\ \frac{P_{max,lv}-P_{sa}}{R_{lv}} & \text{if } P_{max,lv} > P_{sa}, \end{cases}
\end{aligned} \tag{19}$$

where the pressures and resistance in the ventricles are given by

$$\begin{aligned}
R_{lv} &= k_{r,lv} P_{max,lv} \\
P_{lv} &= P_{max,lv} - R_{lv} F_{o,l} \\
R_{rv} &= k_{r,rv} P_{max,rv} \\
P_{rv} &= P_{max,rv} - R_{rv} F_{o,r} \\
P_{max,lv}(t) &= \phi(t) E_{max}(V_{lv} - V_{u,lv}) + [1 - \phi(t)] P_{0,lv}(\exp(k_{E,lv} V_{lv}) - 1) \\
P_{max,rv}(t) &= \phi(t) E_{max}(V_{rv} - V_{u,rv}) + [1 - \phi(t)] P_{0,rv}(\exp(k_{E,rv} V_{rv}) - 1).
\end{aligned} \tag{20}$$

The parameter E_{max} is the ventricle elasticity at the instant of maximal contraction, and V_u is the unstressed ventricle volume. The constants k_r and k_E describe the ventricle resistance and the end-diastolic pressure–volume relationship for the heart. Parameter values that were not explicitly given by the AMC cardiologists were taken from [12].

The heart is activated by the ventricle activation function

$$\phi(t) = \begin{cases} \sin^2\left[\frac{\pi T(t)}{T_{sys}(t)} u\right] & 0 \le u \le T_{sys}/T \\ 0 & T_{sys}/T \le u \le 1 \end{cases} \tag{21}$$

that steers $P_{max,lv}$, the isometric left ventricle pressure. The ventricle activation function is controlled by the baroreflex control system, which is a highly complex function of the sinus nerves.

For simplicity, we approximate the ventricle activation function by a simple sine function, which is shown in figure 9,

$$\phi(t) = \begin{cases} \sin(2\pi\omega) & 0 \le \sin(2\pi\omega) \\ 0 & \sin(2\pi\omega) < 0, \end{cases} \tag{22}$$

with $\omega = 1.25$ *rad* the signal frequency which corresponds to the cardiac cycle.

The rotary pump is modeled as a tube which creates a pressure difference depending on rotational speed. It is assumed that the aortic valve closes perfectly around the tube, so when the valve is open, blood is allowed to flow through the aorta and the pump, and when the valve is closed, blood is only allowed to flow through the pump. The purge fluid is not modelled explicitly. The corresponding

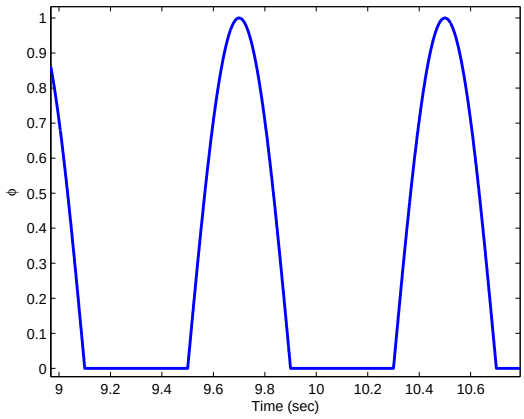


Figure 9: Approximated ventricle activation function ϕ.

equations for the outflow of the left ventricle are therefore modified to

$$F_{o,l} = \begin{cases} \frac{P_{max,lv}+P_{pump}-P_{sa}}{R_{pump}} & \text{if } P_{max,lv} \leq P_{sa} \\ \frac{P_{max,lv}+P_{pump}-P_{sa}}{R_{pump}} + \frac{P_{max,lv}-P_{sa}}{R_{lv}} & \text{if } P_{max,lv} > P_{sa}. \end{cases} \tag{23}$$

The first equation describes flow through the pump when the aortic valve is closed and the second equation describes the flow through the pump and the aorta when the aortic valve is open. The quantity and direction depend on the pressure difference between the left ventricle and the aorta, and on the pressure difference created by the pump. Here, R_{pump} is the flow resistance inside the pump.

As mentioned before, figure 7 shows the flow in l/min for the Impella LP 2.5 pump as a function of the pressure difference over the cannula. The different lines represent different rotational speeds. The exact experimental details were unknown to us, but it is our strong belief that the pressure difference at the horizontal axis is artificially induced. Figure 7 suggests that locally around an operating point a linear relation

$$F = \frac{P_{pump} - \Delta p}{R_{pump}} \tag{24}$$

for the flow through the cannula is reasonable. Here Δp is the artificial pressure difference at the horizontal axis of the experimental data, and R_{pump} is the flow resistance of the pump. We only consider the highest rotational speed of 51000 rpm here. Two data points from the experimental data available during the week (which differed from the ones presented in figure 7) were chosen to determine R_{pump} around the operating point, which resulted in $R_{pump} = 2.25\ s \cdot mmHg/ml$.

6 Numerical Results

The full model was simulated in Matlab with a Runge-Kutta difference scheme with varying timesteps. Figure 10 shows the PV loop (left) and the cardiac output (right) in millilitres per second for a normal patient with and without a pump. The pump pressure was taken to be constant at $P_{pump} = 25\ mmHg$ after calibration based on the experimental data from PV-loop measurements. This means that the pump creates a constant pressure difference of 25 $mmHg$, and therefore also implies that for pressure

differences between the left ventricle and the aorta that are higher than 25 *mmHg*, backflow through the pump arises. Other unknown constants were taken from [12]. We took initial conditions which

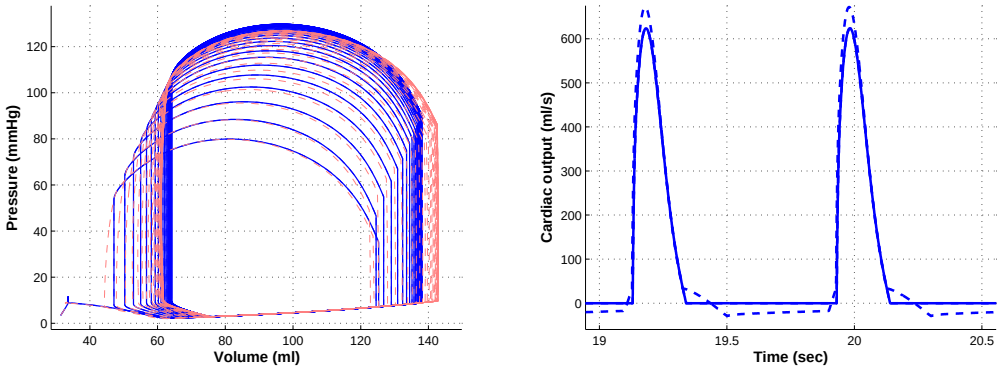


Figure 10: Left: Simulated PV loop. Right: Cardiac output for a normal patient with pump (dash-dotted line) and without pump (solid line). $P_{pump} = 25$ *mmHg*.

differ slightly from the normal operating conditions of the heart to show that the dynamics converge to a stable limit cycle.

In the left plot of figure 10, we observe that starting from some initial value for P_{lv} and V_{lv}, the cardiovascular system stabilizes, and the PV loop converges to a steady cycle. The PV loop of the patient with pump shows a higher maximal volume, which is caused by backflow through the cannula during relaxation. The constant suction induced by the pump also allows less pressure buildup inside the left ventricle at the end of the systolic phase. The right plot of figure 10 shows that the peaks are higher for the patient with the pump, because the pump induces extra outflow during contraction. This increases the cardiac output. On the other hand, during relaxation there is a small backflow through the pump, decreasing the overall cardiac output, since the pressure P_{pump} is too small here to prevent backflow during relaxation.

We also simulated the system with a higher pump pressure of P_{pump}=160 *mmHg*. Figure 11 shows experimental data for the PV loop of the heart of a patient with coronary artery disease, with and without the Impella LP 2.5 pump at the highest rotational speed. The left plot of figure 12 shows that for a normal heart the maximal volume of the left ventricle and the bottom left corner in the PV loop are shifted to the left after insertion of the pump, in correspondence to figure 11. The upper arcs in the PV loop of the simulated normal heart are absent in the measured PV loop of the weak heart. The right plot of figure 12 shows that there is no backflow anymore, and due to the constant outflow, the left ventricle volume is smaller at the end of the systolic phase, which results in a smaller peak in the cardiac output during contraction.

The area below the cardiac output graph equals the total cardiac output in *ml*, and a comparison of figures 10 and 12 shows that the total cardiac output has increased when a stronger pump is used. More specifically, the pump caused an increase from 73.9 *ml* in one cardiac cycle of 0.8 seconds without the pump to 79.3 *ml* when the pump was used, corresponding to an increase from 5.54 litres per minute to 5.95 litres per minute. This increase of approximately 7% is the result of almost 1.5 litres per minute extra cardiac output during the relaxation phase, and roughly 1 litre per minute less cardiac output during the contraction phase.

Further increase of P_{pump} to $P_{pump} = 1000$ *mmHg* gives negative volumes in the PV loop, which

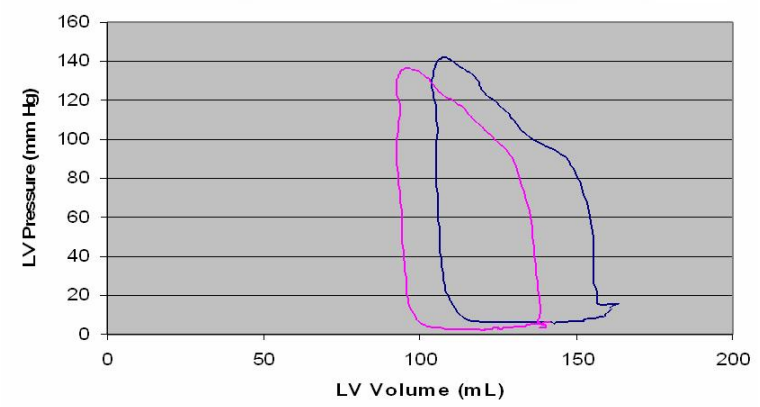


Figure 11: PV loop of the heart of a patient with coronary artery disease, with and without a pump (Measurements provided by AMC).

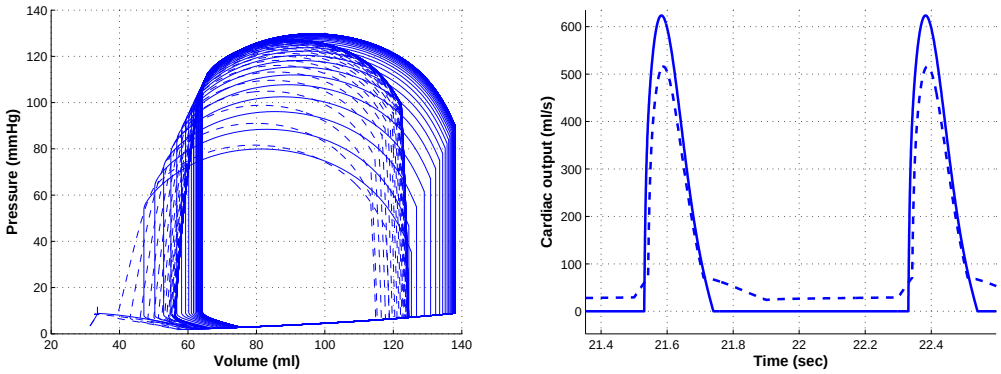


Figure 12: Left: PV loop, and Right: Cardiac output for a normal patient with pump (dash-dotted line) and without pump (solid line). $P_{pump} = 160\ mmHg$.

can be explained as follows. Due to the linearity of the differential equations, state variables are allowed to be negative. The resistances, compliances, and inertances are either measured for normal blood flow or calibrated to fit the model, and therefore the model is only realistic whenever the state variables do not deviate too much from the values of a normal blood flow.

7 Conclusions and Suggestions for Further Research

In this project, we have modeled the effect of a rotary blood pump on the behaviour of a cardiovascular system. It turns out that the resistance encountered by the blood flowing through the pump is a very important design parameter when one tries to calibrate such a model to existing experimental results for the pressure–volume relationships. By varying the pressures generated by the pump, we were able to see phenomena such as backflow through the pump in our simulations.

Under normal operating conditions we found an increase in the cardiac output by 7% as a result of the pump, which is the net result of a substantial increase in output during the relaxation phase

but also a substantial decrease in output during the contraction phase.

There is obvious room for more complex models in future research. We believe that such models should not necessarily involve more detailed modeling of the environment such as the artery systems, since its dynamics seem to be captured quite well, but should rather investigate the exact relationship between pressure and flow through the pump under more extreme circumstances. The design specification of the pump that we investigated during this project focusses on maintaining a constant rotational speed but one might easily envisage control systems for the pump in which other specifications are formulated, to enable an even more beneficial contribution from the mechanical device.

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