Isolating and correcting errors while auditing accounts

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Abstract. A practice in auditing of isolating and correcting errors as part of a statistical audit is introduced with an example. A brief literature review is presented. Decision procedures based on (Poisson) upper confidence bounds are analyzed, using the all-or-nothing model. Three situations are analyzed in detail, with primary focus on the probability that an approved account (still) contains a material error. This probability should not exceed the specified level (0.05 in this paper). It is found that in all these three cases it does, and, for some parameter values, substantially.

1. Introduction and problem statement

One of the problems presented to the 48th European Study Group *Mathematics with Industry* was posed by employees of the Netherlands Court of Audit (de Algemene Rekenkamer) and pertains to the auditing practice of isolating and correcting errors while auditing accounts. A long-standing dispute on the admissibility of some or all of these practices exists between the court and several departmental audit departments (departementale accountantsdiensten). The authors spent the week working on this problem; their findings are reported here. The limited time implies that not all angles could be covered, and many issues that were raised await further study, in particular whether the model adequately reflects common practice.

We start with an example. In a monetary unit sample, an auditor samples 100 Euros from an account of € 1 million. The *interval* J is the number of Euros in the account represented by one Euro in the sample, in this case $J =$ € 1 million/100 = € 10 000. The *materiality*—setting an upper bound to the total error amount that is still acceptable—is set at 4% or € 40 000. As it happens, one error is found, in a 'personnel'

item. Common (statistical) procedure applied here is to compute an upper confidence bound for the total error, based on the Poisson distribution. With one error this bound would be 4.75 times the interval J, or € 47 500. As this upper bound exceeds the materiality, approval would have to be withheld.
It happens to be the case, however, that this error could only occur in a 'personnel' item, and not in other items. Let us say, these are all 'materials' items. The auditor may decide to extrapolate (or project) this error only to the corresponding segment of the account; we shall employ the term *stratum* for such a segment. Furthermore, he may attempt to correct the error.
So, in hindsight, the account and the sample are viewed as follows:

Stratum:	Materials	Personnel
Total amount:	800 000	200 000
Number sampled:	80	20

In the materials stratum no errors are encountered, the corresponding Poisson upper bound therefore is $3J =$ € 30 000. In the personnel stratum one error of size € 5000 is found among the sampled items and it cannot be corrected. An exhaustive examination of the whole stratum follows, but no other errors are found, whence the total error in this stratum is known to be € 5000.
The auditor may now combine the results in an overall upper bound € 30, 000+€ 5, 000 = € 35, 000 that is below the materiality (€ 40, 000). Based on this, the account could be approved. In case the error in the personnel item *could* be corrected, the adjusted bound would be € 30, 000, with the same result.

The question we consider is: *Is it permissible to modify regular statistical procedures with isolation and correction steps? That is, do these procedures retain their nominal properties such as confidence level and significance level?*

2. Isolating and correcting errors

For the sake of clarity we summarize what we mean by 'isolating and correcting errors' in this paper. In reality the range of these practices may be wider, which may cause our conclusions to be conservative. We believe that the following is in agreement with *International Standard on Auditing* 530 [**5**].
What does it mean to *isolate an error*? It should be noted that some errors can only occur in certain strata: the error in the example above could only occur in a personnel item. Isolating an error means the identification of a stratum for which the error is typical and extrapolate the error only to this stratum, rather than to the whole account.

A crucial issue here is whether strata and their boundaries are identified before or after the sample is drawn. In the case after, it may be possible to find a suitable stratum for every error found, and with some creative reasoning perhaps a suitably *small* stratum. After all, extrapolating an error to a small stratum leads to a smaller overall error estimate than when extrapolating it to the whole account.

At this point a modelling difficulty is encountered: how should one model an auditor who isolates and corrects 'in good faith' (so not 'too creatively')? We have tried to resolve this issue by exploring several situations, including two extremes: in Section 6 the strata are identified before the sampling and in Section 7 we assume that every error is isolated to a very small stratum.

Isolating errors may be combined with correcting them. It is natural to attempt to correct errors that are encountered and check for similar errors in similar book items and correct them as well. In the examples, we shall assume that the isolating stratum, when it is identified, is examined exhaustively and the errors found are corrected as much as possible. As a result, the exact error in this part of the account is known. While it is clear that these steps reduce the overall error amount, it is less clear how previously computed confidence bounds should be adjusted, or what these adjusted bounds mean.

3. A brief review of (some of) the literature

There are two papers—[**2**] and [**6**]—that discuss the theoretical aspects of isolating and projecting errors, and that report on the behavior of auditors when confronted with 'unique' errors, i.e., errors that are considered atypical for the population where the sample is taken from. Although these two papers—and also the papers [**3**] and [**4**]—report similar behavior of auditors ("a large majority of auditors favored isolating errors that appeared to be unique" ([**2**], p. 246)), the theoretical parts of these two papers express strongly opposing views.

The oldest of these two papers, Burgstahler and Jiambalvo [**2**], discusses a simple balls-in-urn example to show that "to 'isolate' some sample items found to be in error where 'isolated' items are not projected to the population [. . .] may lead auditors to systematically underestimate population error and result in excessive probability of incorrect acceptance" ([**2**], p. 234). In their balls-in-urn example Burgstahler and Jiambalvo only substantiate the first remark. In fact, their point of view is in part philosophical. In their view, "a fundamental assumption underlying audit sampling is that items are, for sampling purposes, homogeneous in the sense that observation of some subset of items is useful for drawing conclusions about the remainder of the whole population. [. . .] Further, this assumption is necessary for both statistical

and judgemental sampling in auditing." To isolate errors, and not projecting them seems to be in contradiction with this basic assumption of statistical auditing.
Their example is as follows. An urn contains black and red balls; the red balls correspond to items containing an error. The auditor is to construct the maximum likelihood estimate of the number of red balls in the urn. The maximum likelihood estimate is used in [**2**] because it is more intuitive than the more common confidence interval. Suppose there are N balls in the urn, of which an unknown number R are red, and the remaining $N-R$ are black. The number of red balls in a sample of size n drawn with replacement from this population of N balls has a binomial distribution with parameters n and R/N. The maximum likelihood estimate of R/N is r/n, where r is the number of red balls in the sample. This estimator is unbiased, an observation not made in [**2**]. Now suppose that the auditor draws r red balls, $n-r-1$ black balls, and one red cube (a 'unique' error). So one of the sample items is qualitatively different from the others. According to Burgstahler and Jiambalvo there are (at least) two approaches that might be adopted.
In the first approach one simply estimates the number of red items in the urn. This approach treats the red cube the same as a red ball; it is not isolated. The estimated number of red items in the population is $N(r+1)/n$.
In the second approach the red cube is assumed to be unique, and is not taken as an indication that other red items (such as cones, discs, etc.) might be in the urn. In this case it is assumed that of the items in the urn, one is known to be a red cube, an unknown number R of the balls are red, and the remaining $N-R-1$ items are assumed to be black balls. So the sample proportion of red balls is $r/(n-1)$, and the estimate of the total number of red items is $1+(N-1)r/(n-1)$.
Which is the correct approach? According to Burgstahler and Jiambalvo the problem with the second approach is that "auditors are seldom faced with situations where it is reasonable to rule out the possibility that a population contains other unique red shapes. [...] In auditing, no two errors are truly identical; each error is associated with *some* unique characteristic." ([**2**], p. 236).

According to Wheeler *et al.* ([**6**]), Burgstahler and Jiambalvo "suggested that an estimator in which no sample errors are isolated from the estimator (project all sample errors) is normative per standards of statistical inference" ... and "that an estimator biased by the isolation of nonrecurring errors violates those standards." ([**6**], p. 263). In the view of Wheeler *et al.* "a focus on bias ignores the dispersion of estimates." It is shown in [**6**], in an balls-in-urn example that in simplicity resembles that of Burgstahler and Jiambalvo, that in a situation where there are very few red objects other than red balls, the biased estimator giving an estimate of the proportion of red balls in the population might be more desirable than the (unbiased) estimator giving an estimate of the number of red items in the population. It is shown that under certain circumstances the biased estimator has a smaller *mean squared error* than the unbiased estimator (the mean squared error equals the square of the bias plus the variance of the estimator). "A biased estimator may be more precise than an unbiased one" and evidence is provided that "the exclusion of unique errors in developing a sample estimator can increase the accuracy of the estimation process" ([**6**], p. 273).

Apparently, Wheeler *et al.* think to have refuted Burgstahler and Jiambalvo. We disagree. Furthermore, we believe that the examples presented in this paper substantiate Burgstahler and Jiambalvo's claim that isolating and correcting errors may lead to excessive probability of incorrect acceptance.

4. Some terminology and model assumptions

We summarize some terminology and concepts that can be found in standard textbooks on audit sampling techniques, for example, [**1**]. We shall denote the total book value of the account by M, its unknown error fraction by p, and refer to Mp as the overall error (before correction). In formulas the materiality is denoted by mat; in the examples we used $mat = 0.04M$, that is, 4%. Samples are of size n, and items are sampled proportional to size in Euros (a so-called *monetary unit sample*, a standard auditing practice). For the analysis the all-or-nothing principle is used: this is simplest to model, yet considered sufficient for the investigation at hand (see also Section 9). As a consequence the situation can be considered as if a sample of n Euros is drawn from a population of M Euros that contain a fraction p of 'bad' Euros. Let $J = M/n$; this is the *interval*: the amount represented by one sampled Euro.

Confidence bounds used are 95% upper confidence bounds based on the Poisson approximation to the binomial distribution:

Number of errors in the sample:	0	1	2
Confidence bound on total error:	$3J$	$4.75J$	$6.3J$

Taking $mat = 0.04M$ and the confidence level of 95% are fairly arbitrary choices that are immaterial for the patterns that emerge from the results.

5. Our approach

We consider the auditing process as a testing procedure with null hypothesis "overall error *does* exceed the materiality." This test may be performed at significance level 5% by computing a 95% upper confidence bound for the overall error and comparing it with the materiality, rejecting the null hypothesis if the bound falls below. In this way a type I error—approving an account with a material error in it—occurs when a sample from such an account yields a confidence bound below the materiality.

An auditor may (and in practice often does) choose the sample size n by solving $3J = mat$, leading to $n = 3M/mat$. If the sample turns out to be free of errors, the upper bound equals the materiality and the account may be approved. If the auditor wants to be able to tolerate one error, then $n = 4.75M/mat$ is chosen.

When isolation and correction is added to this procedure, an account with a material error in it gets a *second chance to slip through.* Its first chance: produce no errors in the sample; this happens with probability at most 5 percent (we note that for an account whose error just exceeds the materiality and a minimal sample size as described above, this probability equals 5 percent). The second chance: produce a pattern of errors whose isolation and correction produces an upper bound below the materiality but is insufficient to bring the overall error below the materiality.

While this reasoning shows that isolation and correction are wrong in the sense that the nominal type I error probability is exceeded, the practical question is by how much this probability is exceeded and whether it can get 'really bad.'

We took as our assignment to find examples where the actual type I error probability *substantially exceeds* its nominal value. We have constructed situations where we can

(1) model what an accountant might do, and
(2) show P(*(corrected) account is approved with material error in it*) $\geq$ 0.05.

A modelling difficulty arose in connection with the 'strata': the segmentation of the account into strata depends on what kind of errors are discovered in the sample. We found it very difficult to describe this in a suitably general model that would permit a probabilistic analysis. We choose to pretend that there exists some stratification of the account (that is, before the sampling is done) and that examination of sampled 'bad' items uncovers (some of) these strata (in Section 6 the strata are known beforehand).

If the account is divided into k strata, we denote their respective book amounts, error fractions and sample sizes by M_i, p_i and n_i, $i = 1, \ldots, k$. They satisfy the relations $M_1 + \cdots + M_k = M$, $M_1 p_1 + \cdots + M_k p_k = Mp$ and $n_1 + \cdots + n_k = n$. In examples most of these parameters will vary, but the sample will always be homogeneously divided among the

strata, i.e., the sampling intervals are all equal: $M_i/n_i = J_i = J$, for all $i = 1, \dots, k$.
In the next three sections examples are presented of accounts combined with auditing steps that involve isolation and correction. The main difference is in the liberties available and/or taken in this process. In each case we compute the probability of approving an account whose total error (when applicable: after correction) exceeds the materiality.

6. Pre-stratification, errors cannot be corrected

Consider an account with two strata that are delineated *before* the sampling takes place—one may think of 'personnel' and 'materials'. The book sizes are M_1 and M_2, the error fractions p_1 and p_2. So $M = M_1 + M_2$ and the overall error is $Mp = M_1p_1 + M_2p_2$. Errors can only be identified, but not corrected.
The following auditing procedure is followed. A sample of $n = n_1 + n_2$ items is taken, with $n \geq 3M/mat$. If no errors are found, the account is approved. If one or more errors are found, the strata are considered separately. For an error-free stratum the Poisson upper bound is computed: $3M_i/n_i$. A stratum with errors is examined exhaustively and the value of the true error M_ip_i is determined. The (bounds on the) errors are combined by adding them. If this combined bound is below the materiality, then the account is approved.
Our main interest is in the probability of approval when the true error exceeds the materiality, hence we assume $M_1p_1 + M_2p_2 \geq mat$. Let $z_i = (1 - p_i)^{n_i}$, the probability that no errors are found in stratum i. We find:

$$\begin{aligned} \mathrm{P}(approval) = z_1z_2 &+ z_1(1 - z_2)1_{[3M_1/n_1 + M_2p_2 \leq mat]} \\ &+ z_2(1 - z_1)1_{[3M_2/n_2 + M_1p_1 \leq mat]}, \end{aligned}$$

where $1_{[A]} = 1$ when condition A is fulfilled and $1_{[A]} = 0$ otherwise. Intuition suggests that this approval probability should be a decreasing function of both p_1 and p_2. We rewrite the previous expression as the sum of two terms:

$$\begin{aligned} \mathrm{P}(approval) = {} & z_1(1 - z_2)1_{[3M_1/n_1 + M_2p_2 < mat]} \\ & + z_2\left(1_{[3M_2/n_2 + M_1p_1 < mat]} + z_1 1_{[3M_2/n_2 + M_1p_1 \geq mat]}\right). \end{aligned}$$

The first term is decreasing in p_1, since z_1 is a decreasing function of p_1. The sum enclosed by square brackets may equal 1 for small values of p_1, and for larger values it equals z_1, hence is a decreasing function of p_1 as well. Thus, we have shown that P(*approval*) is a decreasing function of p_1 and, by symmetry, also of p_2. Hence, the largest possible approval probabilities occur when the true error $M_1p_1 + M_2p_2$ equals the materiality.

When $n = 3M/mat$ the assumption $\frac{M_i}{n_i} = \frac{M}{n}$ implies that $M_1p_1 + M_2p_2 = mat$ is equivalent with $n_1p_1 + n_2p_2 = 3$. Along this border segment of cases $\mathrm{P}(approval) = z_1z_2$. (Strictly speaking, if p_1 or p_2 is zero we get an extra term; for example, if $p_2 = 0$ the result is $\mathrm{P}(approval) = z_1$; but this corresponds with the limit of $\mathrm{P}(approval) = z_1z_2$ for p_2 going to zero, hence z_2 to 1). This shows that to find the highest approval probability, we should maximize

$$z_1z_2 = (1-p_1)^{n_1}(1-p_2)^{n_2}$$

under the conditions

$$n_1p_1 + n_2p_2 = 3, \quad p_1 \geq 0, p_2 \geq 0.$$

The maximum cannot exceed 0.05 since

$$(1-p_1)^{n_1}(1-p_2)^{n_2} \leq (1-p)^n \leq \mathrm{e}^{-np} = \mathrm{e}^{-3} \leq 0.05,$$

where the first inequality follows by taking logarithms and then applying Jensen's inequality (note that $np = n_1p_1 + n_2p_2$, since $p = mat/M$), and the second from $1 - x \leq \mathrm{e}^{-x}$. The bound 0.05 is attained for $p_1 = p_2 = p$; when p_1 and p_2 are very different the probability is much smaller than 0.05.
When $3M/mat < n \leq 6M/mat$, a similar analysis shows that the approval probability stays below 0.05. For $n > 6M/mat$ it can be exceeded.

In the case of $k \geq 2$ strata, assuming that $Mp \geq mat$, we see that approval occurs when

$$3J + \sum_{i\text{:error found in stratum } i} M_ip_i < mat,$$

where the first term is the error bound for the aggregation of strata where no errors were found, and the second the actual total error for the others. We now restrict ourselves to the special case of 'homogeneous' strata which, admittedly, appears to be the worst case: $p_i = p$, $M_i = M/k$, $i = 1, \ldots, k$. Substituting this we find that approval occurs when the number of strata with errors does not exceed:

$$\frac{mat - 3J}{Mp/k}.$$

Hence, if $mat = 3J$, approval only occurs when no errors are found, and the type I error probability is at most 0.05. This is still the case when the gap $mat - 3J$ is positive but below Mp/k, which is the total error *per stratum.*
However, if the sample size is chosen larger than the minimum $3M/mat$, approval occurs when errors are found in one or more strata. Table 1 lists the probability as a function of the number of strata k. The parameters are $p = 0.05$ and n is chosen to satisfy $mat = 4.75M/n$ as

closely as possible, taking into account that the number sampled per stratum, $n_s = n/k$, should be an integer. Figure 1 shows a plot of these results. Note that in all cases a material error remains, since errors found cannot be corrected. It is seen that the probability of incorrect approval quickly and considerably exceeds acceptable levels as the number of strata increases.

TABLE 1. Probability of incorrect approval for $2, \dots, 19$ strata; $p = 0.05$.

k	n_s	n	P(*approval*)	k	n_s	n	P(*approval*)
2	60	120	0.002	11	11	121	0.228
3	40	120	0.002	12	10	120	0.223
4	30	120	0.033	13	10	130	0.166
5	24	120	0.028	14	9	126	0.363
6	20	120	0.025	15	8	120	0.394
7	17	119	0.115	16	8	128	0.329
8	15	120	0.102	17	7	119	0.591
9	14	126	0.078	18	7	126	0.528
10	12	120	0.089	19	7	133	0.468

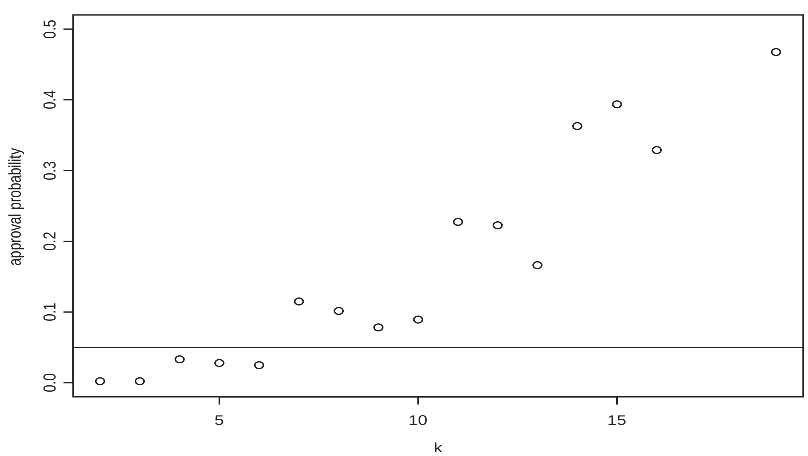

FIGURE 1. Probability of incorrect approval as a function of the number of strata k; $p = 0.05$; horizontal line marks 0.05.

7. Pre-stratification, no correction, dependence on p

In the previous section the situation was analyzed where the number of strata in the pre-stratification varied. In this section we consider the

dependence on the error fraction p. Suppose an auditor judges every error found to be so unique that no others of its kind can be imagined. In effect, this implies that every item is in a stratum on its own and, consequently, there is a large number of strata. Suppose r errors are found in the sample, and error i corresponds to an item of size s_i which after exhaustive examination reveals b_i 'bad' Euros.
Then, considering the part of the sample not contained in the above examined strata as an errorfree sample of $n - r$ from the rest of the account the following (95%) upper confidence bound may be proposed:

$$3\,\frac{M - \sum s_i}{n - r} + \sum b_i$$

For example, let us consider an account with $M = €\,1\,000\,000$ and $k = 1000$ strata, each of size €1000. We use $n = 4M/mat = 100$ and each stratum has the same error fraction p. Then the number of fully examined strata S is approximately binomially distributed with parameters n and p; some strata may be sampled more than once, so the number of fully examined strata is stochastically smaller than this binomial approximation. The actual acceptance probability therefore is bounded below by

$$\mathrm{P}\left(3\frac{M - SM/k}{n - S} + Sp\frac{M}{k} < mat\right).$$

Figure 2 shows a plot of this probability, which is close to 1 for $p \leq 0.11$ and drops below 0.05 only when $p > 0.244$. We see that this case, which could be labeled 'extreme isolation,' produces extremely high probabilities of incorrect approval.

8. Homogenous strata, every error can be corrected

Until now we have considered examples without possibility of correction of errors. In this section we consider the situation where *every* error can be corrected. Suppose an account consists of k homogenous strata of equal size: $p_i = p$, $M_i = M/k$, $i = 1, \ldots, k$. We choose $p = \frac{k}{k-1} \cdot \frac{mat}{M}$, so that correcting one whole stratum still leaves enough errors in the $k - 1$ remaining to attain materiality. This implies the following:

- if zero errors are found in the sample, the account is approved;
- if all errors lie in one stratum, that stratum will be corrected and the resulting account is approved;
- if more than one stratum is corrected the remaining error drops below the materiality.

Let S be the number of strata in which errors are detected. Then S has a binomial distribution with parameters k and $\pi = 1 - (1 - p)^{n_s}$, where

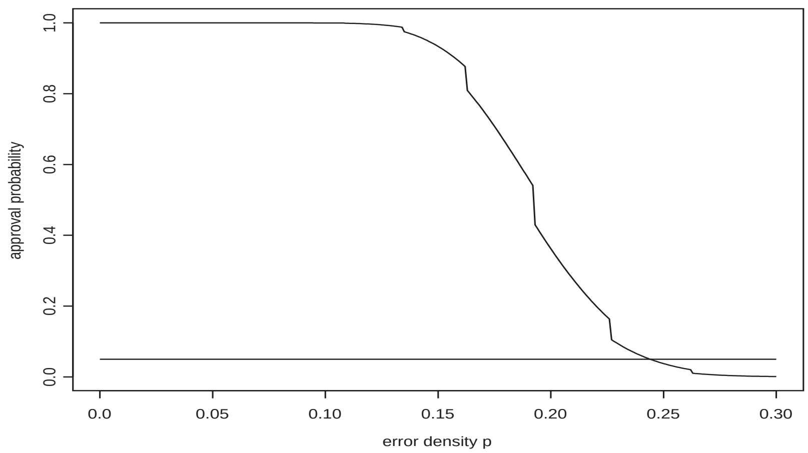

Figure 2. Approval probability as a function of the error fraction p; horizontal line marks 0.05.

n_s is the number of items sampled per stratum, equal to n/k rounded upward. It follows that

$$\mathrm{P}(\textit{incorrect approval}) = \mathrm{P}(S \leq 1)\,.$$

We have chosen $n \approx 100$, that is, $n \approx 4M/mat$, assuming a sample size slightly above the minimum. In Table 2 and Figure 3 the results are presented. While the results may look less than dramatic, we remark that for the minimum sample size $n = 75$ approval probabilities are larger: for $k = 3, \ldots, 19$, it varies from 0.11 to 0.18. Also, it is suspected that similar examples could be constructed with even larger approval probabilities.

Table 2. Probability of incorrect approval for $2, \ldots, 19$ strata; $p = 0.04 \cdot k/(k-1)$.

k	n_s	n	p	P(*approval*)	k	n_s	n	p	P(*approval*)
2	50	100	0.080	0.031	11	10	110	0.044	0.051
3	34	102	0.060	0.041	12	9	108	0.044	0.056
4	25	100	0.053	0.053	13	8	104	0.043	0.065
5	20	100	0.050	0.059	14	8	112	0.043	0.050
6	17	102	0.048	0.059	15	7	105	0.043	0.064
7	15	105	0.047	0.055	16	7	112	0.043	0.051
8	13	104	0.046	0.059	17	6	102	0.042	0.072
9	12	108	0.045	0.053	18	6	108	0.042	0.059
10	10	100	0.044	0.072	19	6	114	0.042	0.048

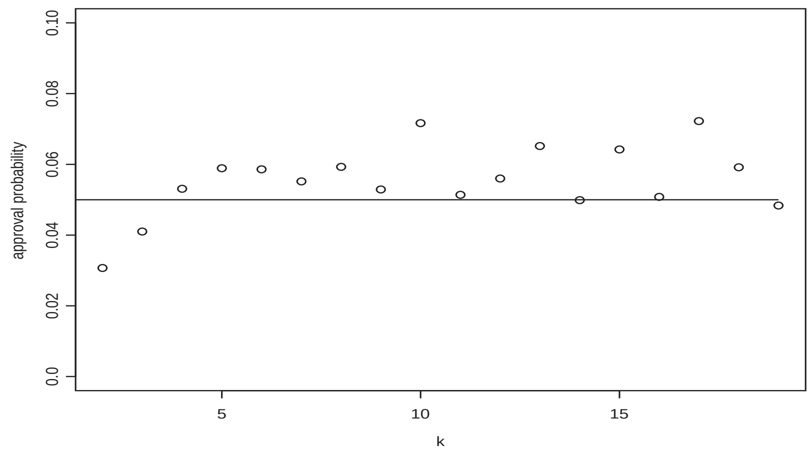

FIGURE 3. Probability of incorrect approval as a function of the number of strata k; $p = 0.04 \cdot k/(k-1)$; horizontal line marks 0.05.

9. Conclusions and final remarks

We briefly discussed two papers dealing with isolating and correcting errors. Burgstahler and Jiambalvo [**2**] predict that isolation and correction may lead to very high probabilities of incorrect approval, whereas Wheeler *et al.* ([**6**]) hold an opposite point of view.
It seems to be difficult to model the isolation and correction procedure used in the auditing process. In order to formulate and analyse a model for this procedure, we have considered three examples with prestratification. For these examples we analysed the effect on the probability of incorrect approval of the number of strata, the error fraction and the possible corrections.
These examples seem to show that in many cases the probability of incorrect approval is (much) larger than the allowed margin, supporting the view of Burgstahler and Jiambalvo.
Using so-called Stringer bounds will not amend this situation: we believe that similar examples with similar excessively high approval probabilities can be constructed. We conclude that reconsideration of the standards pertaining to isolation and correction, as formulated in the *International Standard on Auditing 530*, seems to be in order.

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