ADR Option Trading

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1. Introduction

A company that is seeking to raise capital to finance necessary investments, can issue stocks, which are basically certificates of partial ownership in the company. There are many rules for issuing stocks, one of which is that the company has its seat in the country where the stocks are issued. If, nevertheless, a non-US company, like Royal Dutch N.V., wants to raise capital in the US, it can issue ADRs. ADR is an acronym for American Depository Receipt, which is a certificate issued by a US bank, representing a certain amount of stock of a non-US company on a non-US exchange. Just as US stock, ADRs can be traded, cleared and settled on American exchanges in accordance with US market regulations. ADRs are US securities and are quoted and traded in US dollars. This makes it easier for Americans to invest in non-US companies, due to the widespread availability of dollar-denominated price information, lower transaction costs, and timely dividend distributions. The price of an ADR follows, accounting for the currency exchange rate, more or less the price in the home country; if the US price gets too far off from the price in the home country, arbitrageurs will step in the market and the arbitrage opportunity will soon cease to exist. In order to provide the American investor with more investment possibilities, options are issued on these ADRs. These ADR options are also listed on US markets, denominated in US dollars and also the strike is specified in US dollars.

Non-US market makers trading options on a stock listed in their domestic country might be interested in adding the corresponding ADR options to their portfolio. The interesting part of ADR option trading, is the integration of the position in these US listed options with the domestic option position. The advantage of this integration is that we have - from a risk-management point of view - a clear perspective of the exposure the market maker has with respect to a single stock. If we consider for example stocks Royal Dutch (RD), traded in Amsterdam and their corresponding US ADRs, we can - once we are able to manage this as one integrated position - compute a single delta, gamma or vega for our Royal Dutch position. Furthermore we would like to exploit the mis-pricing of US options with respect to their Dutch counterparts and so we need a pricing model to price the foreign US dollar denominated ADR options consistently with the domestic Euro denominated options and stock.

The market model. We start building our market model from the domestic stock price process $\{S_t\}_{t\geq 0}$, that we model as a Geometric Brownian Motion,

$$(1)\qquad \frac{dS_t}{S_t} = \mu dt + \sigma dW_t \quad S_0 = s_0 \quad \Leftrightarrow \quad S_t = s_0 e^{(\mu - 0.5\sigma^2)t + \sigma W_t}.$$

This is the classical approach to stock price modeling as is also used by [**2**]. For the Euro/Dollar exchange rate process $\{FX_t\}_{t\geq 0}$ we also assume that it is given by a GBM,

$$(2)\qquad \frac{dFX_t}{FX_t} = \alpha dt + \Sigma_1 dW_t + \Sigma_2 dZ_t \quad FX_0 = f_0 \quad \Leftrightarrow$$
$$FX_t = f_0 e^{(r - 0.5(\Sigma_1 + \Sigma_2)^2)t + \Sigma_1 W_t + \Sigma_2 Z_t}.$$

Here we used another standard Brownian Motion Z independent of W to model a dependence structure between the domestic asset S and the exchange rate FX. This is the same approach as in [**3**] and [**5**]. We remark that the direction of the exchange rate is such that the value FX_t is the number of Euros you have to pay for one US dollar at time t. Denote the ADR stock price process by $\{A_t\}_{t\geq 0}$. We assume that the market is efficient, i.e. arbitrageurs are active to force the following relation to hold,

$$(3)\qquad A_t = \frac{S_t}{FX_t} \quad t \geq 0.$$

This relation is investigated in [**1**] and turned out to be quite accurate looking at real markets where prices are formed concerning transaction and conversion costs. If we consider a European call option with strike K written on the ADR and therefore listed on the foreign market, the pay-off in US dollars Φ_C of this contract can be written using the previous relation by

$$(4)\qquad \Phi_C(A_T) = \Phi_C\left(\frac{S_T}{FX_T}\right) = \left(\frac{S_T}{FX_T} - K\right)^+.$$

We remark that also the strike K is denominated in US dollars. Now we need to find the equivalent martingale measure $\mathbb{Q}$ turning all the assets in our economy into martingales in order to price this derivative. First we have to identify the assets we can use building our portfolio. As in the classical approach we use both the domestic stock S and the domestic bank-account process $\{B_t^{(d)}\}_{t\geq 0}$ as assets in our economy, where $B^{(d)}$ is given by,

$$B_t^{(d)} = e^{r_d t}.$$

Here r_d is the classical risk-free rate. As an extra asset we introduce the foreign or US bank-account process $\{B_t^{(f)}\}_{t\geq 0}$. We are considering our economy from the domestic perspective (Euro-zone), so we should denominate all our assets in the same domestic currency and therefore we consider the US bank-account denominated in Euros as a risky asset. So the US bank account is not a bank account in the classical sense, i.e. from the domestic point of view it is not offering the risk-free rate. To illustrate this we take a closer look at trading of this asset, which is converting one Euro at time $t = 0$ into $(FX_0)^{-1}$ US dollars and deposit this amount on a US bank account. At time t we earned the US risk-free rate on the deposited dollar amount and in order to calculate its value in Euros we have to convert it again by the stochastic exchange rate FX_t. We have for $B^{(f)}$,

$$B_t^{(f)} = \frac{FX_t}{FX_0} e^{r_f t}.$$

Here r_f is the foreign risk-free rate, that we cannot obtain risk-free if we denominate the value in Euros. Using the dynamics of the exchange rate FX_t we can compute the dynamics of $B_t^{(f)}$ by,

$$\begin{aligned} dB_t^{(f)} &= FX_t e^{r_f t} r_f dt + e^{r_f t} dFX_t \\ &= B_t^{(f)} \left[(r_f + \alpha)dt + \Sigma_1 dW_t + \Sigma_2 dZ_t \right] \end{aligned}$$

From (3) we recognize that the ADR price process is completely determined by the domestic stock price process S and the exchange rate FX and therefore we do not want to introduce A as an extra asset in our economy. If we decide to choose $B^{(d)}$ as the numéraire, which is more or less a standard choice, we can find the option price by identifying the equivalent martingale measure $\mathbb{Q}$ such that the discounted asset price processes $S_t[B_t^{(d)}]^{-1}$ and $B_t^{(f)}[B_t^{(d)}]^{-1}$ are martingales. If we denote the discounted stock price process by $\tilde{S}$, we are looking for a measure $\mathbb{Q}$ such that both the process $\tilde{W}$ defined by,

$$\tilde{W}_t = W_t + q_1 t \quad \Leftrightarrow \quad d\tilde{W}_t = dW_t + q_1 dt$$

is a standard Brownian Motion and the discounted stock price process $\tilde{S}$ is a martingale. The existence of such a $\mathbb{Q}$ is guaranteed by the Girsanov Theorem, see e.g. [**4**]. Writing the dynamics of $\tilde{S}$ in terms of $\tilde{W}$ we get,

$$d\tilde{S}_t = \tilde{S}_t \left[(\mu - r_d)dt + \sigma dW_t \right] = \tilde{S}_t \left[(\mu - r_d - \sigma q_1)\, dt + \sigma d\tilde{W}_t \right].$$

For $\tilde{S}$ to be a martingale, we set the drift term equal to zero, so

$$q_1 = \frac{\mu - r_d}{\sigma}.$$

Now we have solved q_1 we directly obtain the dynamics of S under $\mathbb{Q}$ by,

$$\begin{aligned} dS_t &= S_t \left[\mu dt + \sigma dW_t\right] \\ &= S_t \left[\mu dt + \sigma d(\tilde{W}_t - q_1 dt)\right] = S_t \left[r_d dt + \sigma dW_t\right]. \end{aligned}$$

This is not a surprising result, because it is equivalent to the classical risk-neutral Black-Scholes dynamics of the stock price process, see [**2**]. Now we proceed by changing the drift of the other Brownian Motion Z such that both the process $\tilde{Z}$ defined by,

$$\tilde{Z}_t = Zt + q_2 t \quad \Leftrightarrow \quad d\tilde{Z}_t = dZ_t + q_2 dt$$

is a $\mathbb{Q}$ standard Brownian Motion, independent of $\tilde{W}$ and the process $\tilde{B}$ is a martingale. Here $\tilde{B}$ denotes the discounted foreign bank account process $B_t^{(f)}[B_t^{(d)}]^{-1}$. For the dynamics of $\tilde{B}$ we obtain,

$$\begin{aligned} d\tilde{B}_t &= \frac{1}{B_t^{(d)}} dB_t^{(f)} - \frac{\tilde{B}_t}{B_t^{(d)}} dB_t^{(d)} \\ &= \tilde{B}_t \left[(r_f - r_d + \alpha)dt + \Sigma_1 dW_t + \Sigma_2 dZ_t\right] \\ &= \tilde{B}_t \left[(r_f - r_d + \alpha)dt + \Sigma_1(d\tilde{W}_t - q_1 dt) + \Sigma_2(d\tilde{Z}_t - q_2 dt)\right] \\ &= \tilde{B}_t \left[\left(r_f - r_d + \alpha - \Sigma_1 \frac{\mu - r_d}{\sigma} - \Sigma_2 q_2\right) dt + \Sigma_1 d\tilde{W}_t + \Sigma_2 d\tilde{Z}_t\right]. \end{aligned}$$

Again we need the drift term equal to zero for $\tilde{B}$ a $\mathbb{Q}$ martingale, which is satisfied if we put,

$$q_2 = \frac{r_f - r_d + \alpha - \Sigma_1 \frac{\mu - r_d}{\sigma}}{\Sigma_2}.$$

Now we find for the exchange rate process FX under the pricing measure $\mathbb{Q}$:

$$\begin{aligned} dFX_t &= FX_t \left[\alpha dt + \Sigma_1 dW_t + \Sigma_2 dZ_t\right] \\ &= FX_t \left[(\alpha - \Sigma_1 q_1 - \Sigma_2 q_2)dt + \Sigma_1 d\tilde{W}_t + \Sigma_2 d\tilde{Z}_t\right] \\ &= FX_t \left[(r_f - r_d)\, dt + \Sigma_1 d\tilde{W}_t + \Sigma_2 d\tilde{Z}_t\right]. \end{aligned}$$

It is clear that the exchange rate process FX does not enter into our portfolio, because it is not possible to actually buy or sell the exchange rate. We can however keep an amount of foreign currency on a foreign bank account, that is why we decided to take the foreign bank account as a possible asset for our portfolio. We used the fact that all discounted assets - where we used the domestic bank account as numéraire process - have to be martingales under $\mathbb{Q}$ to derive the dynamics of the exchange rate process FX under $\mathbb{Q}$. We can now use this measure and the corresponding asset-dynamics to price all attainable claims, expressed in the domestic currency. Suppose $\{X_t\}_{t\geq 0}$ is the vector containing all our assets in the economy, a claim Φ is attainable if we can find a self-financing strategy ϕ such that,

$$\phi_0 \cdot X_0 + \int_0^T \phi_u dX_u = \phi_T \cdot X_T = \Phi(X_T). \tag{5}$$

Here the self-financing property is used in the first equality sign. We used in our setup the domestic bank account as the numéraire and constructed the corresponding martingale measure. If we consider the discounted asset price process $\tilde{X}$ given by,

$$\tilde{X}_t = \frac{X_t}{B_t^{(d)}},$$

we have that the self-financing property of ϕ also holds for $\tilde{X}$. Now we use that the discounted asset prices are martingales and therefore - assuming the right conditions on ϕ - we have that the stochastic integral representing the gains of the strategy ϕ over time is also a martingale. We remark that we have to impose some conditions on ϕ to guarantee the stochastic integral to be a martingale instead of being a local martingale only. From a no-arbitrage argument we have that $\phi_0 \cdot X_0$ must be equal to the price of the contract at time 0. Now we can compute this price V_Φ of the option with pay-off Φ using the martingale property of the discounted asset prices by,

$$\begin{aligned} V_\Phi &= B_0^{(d)} \phi_0 \cdot \tilde{X}_0 = B_0^{(d)} \mathbb{E}_{\mathbb{Q}} \left[\phi_T \cdot \tilde{X}_T \right] \\ &= \frac{B_0^{(d)}}{B_T^{(d)}} \mathbb{E}_{\mathbb{Q}} \left[\Phi(X_T) \right] = e^{-r_d T} \mathbb{E}_{\mathbb{Q}} \left[\Phi(X_T) \right]. \end{aligned} \tag{6}$$

All the assets in our economy are in Euro currency and as we are using this assets to replicate our foreign option (4), we also have to denote the pay-off of that option in Euros $\tilde{\Phi}_C$ by,

$$\tilde{\Phi}_C(A_T) = FX_T \Phi_C(A_T) = (S_T - FX_T K)^+ . \tag{7}$$

Now we use the pricing formula (6) to obtain $\tilde{C}_{for}$ the price of the foreign option in Euros:

$$\tilde{C}_{for} = e^{-r_d T} \mathbb{E}_{\mathbb{Q}} \left[FX_T \left(A_T - K \right)^+ \right] = e^{-r_d T} \mathbb{E}_{\mathbb{Q}} \left[\left(S_T - FX_T K \right)^+ \right]. \tag{8}$$

As this option is traded in the US market we need an option value in US dollars in order to compare our theoretical price to the market price. This is nothing else then converting the Euro value $\tilde{C}_{for}$ into a value C_{for} in US dollars by dividing by FX_0. In this section we are considering European call options, whereas in practice the options on ADRS are American. A related matter of practical concern is the ex-dividend date of an ADR that is typically a few days earlier or later than the ex-dividend date of the non-US stock the ADR is based on.

Calibration of the Model to the market. If we want to use the model in practice, we should come up with a method to determine the parameters σ, Σ_1 and Σ_2 as they appear in (1) and (2). We will relate these parameters to the volatility of both the stock price process and the exchange rate process, where the volatility is defined as the standard deviation of the log-returns, scaled by the square-root of time up to time units of 1 year. Furthermore we relate the quantities we have to estimate to the correlation between the log-returns of the stock price process and the log-returns of the exchange rate process. The reason for doing so, is that we can obtain the volatilities and correlation as described above directly from an information system (e.g. Bloomberg) as these systems are used in a trading firm.
Suppose we have observations of the price processes at time points $\{t_0, t_1, .., t_M\}$, where we denote the fixed time step $t_{i+1} - t_i$ by Δt. Now we find for the log-return s_i at time t_{i+1} of the stock price process S,

$$s_i = \ln \frac{S_{t_{i+1}}}{S_{t_i}} = (\mu - \frac{1}{2}\sigma^2)\Delta t + \sigma(W_{t_{i+1}} - W_{t_i}) \stackrel{d}{=} (\mu - \frac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t}U, \tag{9}$$

where $U \sim \mathcal{N}(0,1)$. If we denote the volatility estimate we obtain from our information system by $\hat{\sigma}^S$, we immediately can use it as an estimate for our σ. As we choose our domestic stock model as the standard GBM we of course expect that the volatility estimate from the information system is an estimator for our volatility parameter. The more interesting case is the determination of the parameters Σ_1 and Σ_2. For the exchange rate process FX we have a similar expression for the log-returns f_i,

$$f_i = \ln \frac{FX_{t_{i+1}}}{FX_{t_i}} = (\alpha - \frac{1}{2}(\Sigma_1 + \Sigma_2)^2)\Delta t + \Sigma_1(W_{t_{i+1}} - W_{t_i}) + \Sigma_2(Z_{t_{i+1}} - Z_{t_i})$$

$$\stackrel{d}{=} (\alpha - \frac{1}{2}(\Sigma_1 + \Sigma_2)^2)\Delta t + \sqrt{\Delta t}\,(\Sigma_1 U + \Sigma_2 V)\,. \tag{10}$$

Here we have again $V \sim \mathcal{N}(0,1)$, where U and V are independent. If we denote the estimate for the volatility of the exchange rate process by $\hat{\sigma}^{FX}$, we can write,

$$\hat{\sigma}^{FX} = \widehat{\sqrt{\Sigma_1^2 + \Sigma_2^2}}. \tag{11}$$

We need more information to determine Σ_1 and Σ_2 separately. For the covariance between the log returns of the stock price process S and the exchange rate process FX at time t_{i+1} we have,

$$\mathrm{COV}(s_i, f_i) = \sigma\Delta t\mathrm{COV}(U, \Sigma_1 U + \Sigma_2 V) = \sigma\Delta t\Sigma_1.$$

For the correlation ρ we have

$$\rho = \frac{COV(s_i, f_i)}{\sigma\Delta t\sqrt{\Sigma_1^2 + \Sigma_2^2}} = \frac{\Sigma_1}{\sqrt{\Sigma_1^2 + \Sigma_2^2}}. \tag{12}$$

So using these relations we can, provided with estimates $\hat{\sigma^S}$, $\sigma^{\hat{FX}}$ and $\hat{\rho}$ from the information system, come up with estimates for the parameters in our model,

$$\sigma = \widehat{\sigma^S}, \quad \Sigma_1 = \hat{\rho}\widehat{\sigma^{FX}}, \quad \Sigma_2 = \widehat{\sigma^{FX}}\sqrt{(1-\hat{\rho})^2}. \tag{13}$$

Conclusion. In this paper we treat the topic of pricing options on ADRs listed on US markets. The importance is in the fact that the ADR price process is directly related to the price process of the corresponding stock in the non-US market. This relation extends to the possibility to replicate the foreign listed option with instruments in the domestic market. We set up an economy consisting of the domestic stock and bank account and moreover we introduced the foreign bank account as an additional risky asset. After modeling these instruments, we derived an equation for the price of a foreign call option. Finally we showed how we can relate standard estimates obtained from a trading information system to the parameters of our model.

References

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