Modelling of moisture induced warp in panels containing wood fibres

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Abstract

In this contribution the deformation of panels, used a.o. in furniture, is discussed in relation to the moisture content. It is shown how the variations in temperature, water and resin concentrations during the pressing process of the panels can be modelled. The panel deformation is modelled using linear elasticity theory. An explicit analytical expression for the long term behaviour of the water concentration is presented.

Keywords

Panel deformation, Moisture content, Linear elasticity theory.

1 Introduction

Trespa International BV manufactures high quality panel material. This material consists of polymerised resin reinforced with wood fibres or sulphate paper. These panels may deform due to variations in moisture content and the motivation for this study is to obtain mathematical models which help to explain this deformation.

The purpose of this report is to gain a better understanding of the behaviour of TRESPA panels. The modelling concerns the three processes of resin curing during pressing, moisture movement and panel deformation. These three processes depend on each other. The resin curing provides the initial water concentration for the moisture movement model. The moisture distribution itself is an input to the panel deformation model.

The contents will now be outlined. Section 2 describes two mathematical models for the resin curing process. The first model was discussed in a previous study [?]. The second model describes the temperature, concentration of the water and resin during pressing. In Section 3, the movement of water is modelled by a linear diffusion equation. A numerical solution is included. Section 4 derives a new mathematical model for the panel deformation based on linear elasticity. After long time periods the water concentration will be linear across the panel, an analytical solution for the deformation is presented in this case.

2 A thermo-chemical model for resin curing

In this section we briefly describe two models for resin curing during the pressing of TRESPA-panels; a more elaborate description of the first model is given in [?]. During the manufacturing of panels, sheets of resin impregnated paper enclosed by two layers of padded paper are pressed together. A schematic, three-layer model of a panel is given in Figure 1. A panel thus has two polsters of padded paper and a core consisting of impregnated paper. During the pressing of a panel, high temperatures are applied at the boundaries $z = -h$ and $z = h$ of the panel, which causes heating of the panel. This induces a polymerization reaction in the core. Heat is released during the polymerization, which again leads to an increase of temperature in the panel. This process continues until the chemicals in the core are depleted.

2.1 One species model

The curing process can be described by the temperature T in the panel and the concentration C_1 of chemicals involved in the polymerization reaction in the core. To keep the model feasible, we make the following assumptions:

1. The materials are incompressible.
2. Resin curing is a first order reaction.
3. Diffusion of resin is negligible.
4. All variables only depend on the transverse space coordinate z.

The governing equations are now the following. In both polsters the heat equation holds, and in our particular case is given by [?]

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\Big(\lambda \frac{\partial T}{\partial z}\Big), \quad (1)$$

with ρ, c_p and λ the density, specific heat and thermal conductivity, respectively, of the polster material. These variables are assumed to be constant. In the core, heat transport is coupled with the polymerization reaction, and the governing equations read

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\Big(\lambda \frac{\partial T}{\partial z}\Big) + \Delta H k C_1, \quad (2)$$

$$\frac{\partial C_1}{\partial t} = -kC_1, \quad (3)$$

with ΔH and k the enthalpy of polymerization and the reaction rate, respectively. This reaction rate is given by the Arrhenius expression

$$k = A\mathrm{e}^{-E_a/RT}, \quad (4)$$

with A, E_a and R the pre-exponential factor, activation energy and gas constant, respectively.

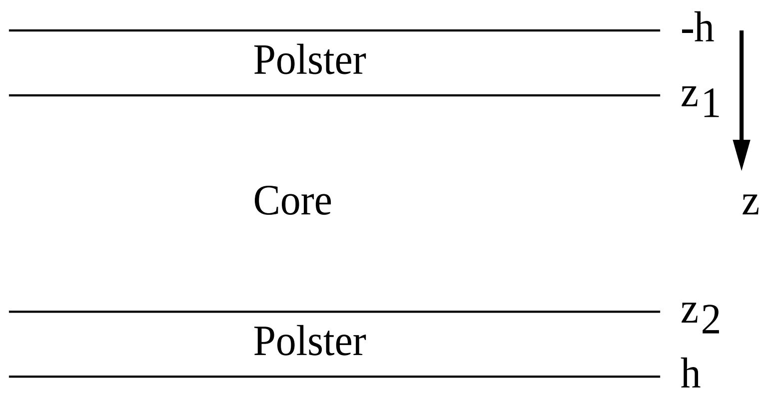

Figure 1: Schematic representation of a three-layer TRESPA-panel.

The model (??)-(??) has to be completed with initial and boundary conditions and conditions at the interfaces between core and polsters. As initial conditions, we choose a constant temperature T and concentration C_1. At the boundaries, the temperature is given as a function of time, i.e.

$$T(-h,t) = T_1(t), \quad T(h,t) = T_2(t), \quad t > 0. \quad (5)$$

Finally, at the interfaces we impose that both the temperature T as well as the heat flux $\lambda \partial T/\partial z$ are continuous. This results in the conditions

$$T(z-,t) = T(z+,t), \qquad \Big(\lambda \frac{\partial T}{\partial z}\Big)(z-,t) = \Big(\lambda \frac{\partial T}{\partial z}\Big)(z+,t), \quad z = z_1, z_2. \tag{6}$$

These conditions mean that there is no accumulation of heat at the interfaces.

As an example, we have computed a numerical solution of the system (**??**)-(**??**) using the finite difference method. More specifically, we used central differences for space discretization and the ϑ-method for time integration [**?**]. For more details, the reader is referred to [**?**]. In this example, we have a constant initial temperature and concentration. Then, at $t = 0$, a high temperature is applied at the boundaries of the panel. The evolution of the temperature and concentration profiles is shown in Figure 2. Initially, the temperature increases due to conduction and heat production and at the same time the concentration decreases. When the chemical species are depleted, the temperature profile tends to the constant steady state.

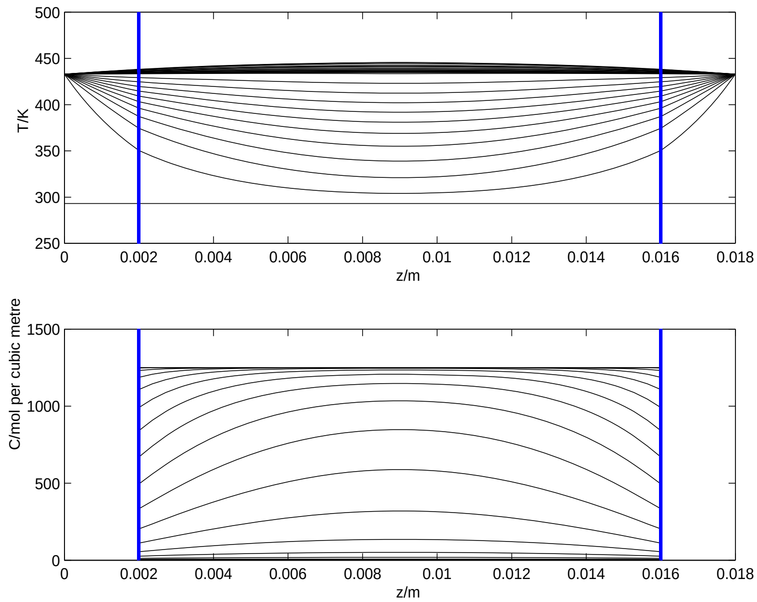

Figure 2: Temperature and concentration profiles in a TRESPA-panel.

2.2 Two species model

A slightly more sophisticated model for resin curing will now be introduced. The dependent variables are the temperature T, the concentration of water C and the concentration of resin C_1. There are now five layers in the model as shown in Figure 3. In the polster and metal regions, the standard heat equation (**??**) applies. In the core, we have (**??**)-(**??**) and

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial z}\Big(D\frac{\partial C}{\partial z}\Big) + \gamma k C_1, \tag{7}$$

where γ is a dimensionless number representing the rate at which moisture is produced relative to the rate at which chemicals involved in the polymerization are reduced. The thermal conductivity is now taken to be of the form

$$\lambda(C_1) = \begin{cases} \lambda_1 & C_1 \le C_{crit}, \\ \lambda_2 & C_1 > C_{crit}, \end{cases}$$

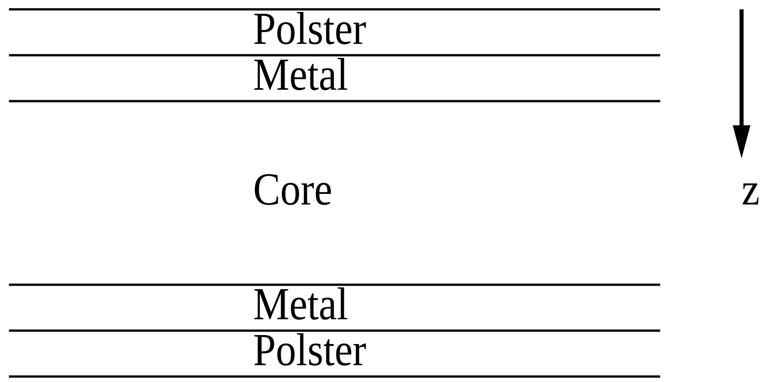

Figure 3: Schematic representation of a five-layer TRESPA-panel.

where C_{crit} is an experimentally observed constant and the diffusivity of water is given by

$$D(T) = B \exp(-T_*/T)$$

where $B = 3 \times 10^{-5} \mathrm{m}^2\mathrm{s}^{-1}$ and $T_* = 5245\mathrm{K}$. The boundary conditions are (**??**) and the interface conditions between the core and metal are given by (**??**) and

$$\frac{\partial C}{\partial z}(z,t) = 0, \quad z = z_1, z_2. \tag{8}$$

3 Moisture transport in panels

In this section we outline a model for the transport of moisture in a panel. A non-uniform water distribution leads to non-uniform stresses in the panel, and this will lead to warping of the panel. This will be described further in the next section.

For moisture transport in panels, we adopt a particularly simple model, viz. we consider the panel as a single layer of material in which diffusion of water takes place. Further assumptions are:

1. Swelling or shrinkage in the transverse direction are negligible.
2. The material is homogeneous.
3. The concentration of water only depends on the transverse space coordinate z.

Under these assumptions, the diffusion of water in a panel is governed by the equation [**?**]

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial z}\Big(D\frac{\partial C}{\partial z}\Big), \tag{9}$$

with C the concentration of water and D the diffusion coefficient. The diffusion coefficient generally depends on the temperature T, but in our model we assume it to be constant. A given initial concentration $C_0(z)$ and Dirichlet boundary conditions complete the problem; we will not specify these any further.

As an example we have computed a numerical solution of (**??**) using central differences for space discretization and the ϑ- method for time integration. The result is presented in Figure 4. This figure typically shows the evolution of concentration profiles starting from a constant initial concentration, when at $t = 0$ the boundaries are exposed to higher concentrations of water, due to moisture in the environment.

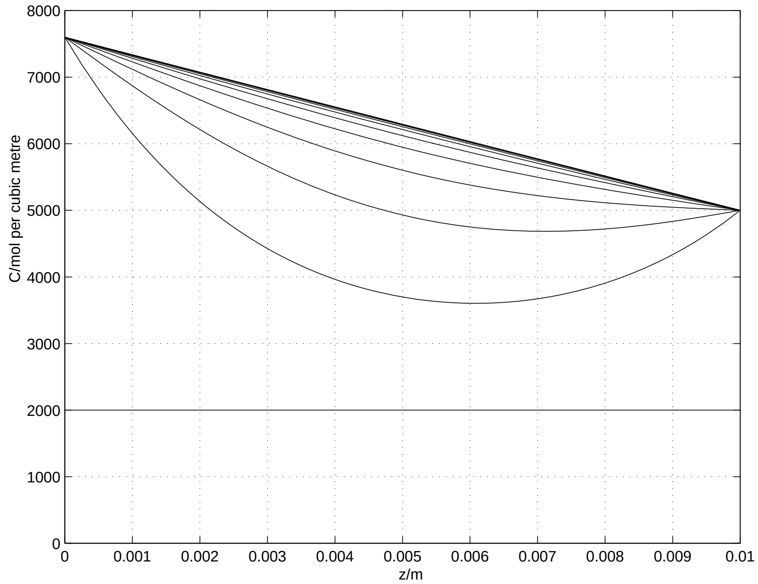

Figure 4: Water concentration profiles in a TRESPA-panel

4 Panel Deformation

In this section the warpage of Trespa panels due to humidity effects is studied. We assume that the Trespa panel can be modelled as a beam. This model is derived under the assumption that the material is linearly elastic [?] and that gravity is negligible.

We may assume that the centre of the beam is clamped due to symmetry considerations. The coordinate system used is sketched in Figure ??. We define the displacement vector $(u_x, u_z) =$

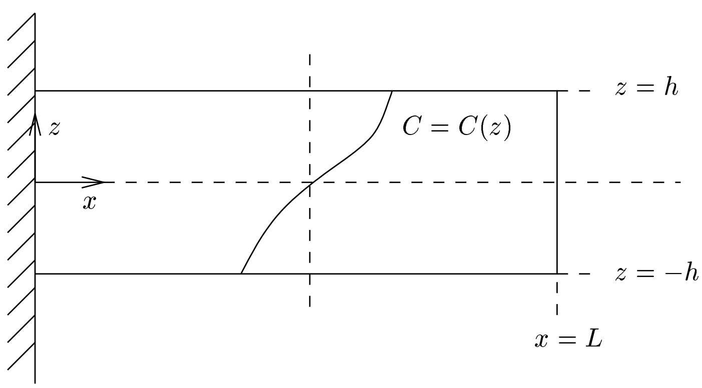

Figure 5: The geometry of the beam model.

(u, w). There are five unknowns in the warpage problem, namely the stresses in the x and z-direction, t_{xx}, t_{xz} and t_{zz}, and the displacements u and w. Therefore, we need five equations to determine these variables.

First of all, conservation of momentum yields

$$\frac{\partial t_{xx}}{\partial x} + \frac{\partial t_{xz}}{\partial z} = 0, \tag{10a}$$

$$\frac{\partial t_{xz}}{\partial x} + \frac{\partial t_{zz}}{\partial z} = 0. \tag{10b}$$

To close the system we need three additional equations, which follow from the constitutive behaviour of the material. We define the deformation tensor as follows

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \qquad i, j = x, z. \tag{11}$$

The deformation tensor is assumed to be split up into a part which represents the deformation caused by elastic effects and a part that represents the deformation caused by expansion of the material due to swelling:

$$e_{ij} = e_{ij}^{(el)} + e_{ij}^{(sw)}. \tag{12}$$

In our model, we assume that the deformation of the panel due to humidity is linearly dependent on the concentration of moisture inside the material. Experiments carried out by Trespa International B.V. support this assumption. This leads to the the following set of equations for $e_{ij}^{(sw)}$

$$e_{xx}^{(sw)} = \alpha_x C, \tag{13a}$$

$$e_{xz}^{(sw)} = 0, \tag{13b}$$

$$e_{zz}^{(sw)} = \alpha_z C, \tag{13c}$$

where α_x and α_z represent the swelling in the x and the z direction, respectively. We note that this model is analogous to thermoelasticity except that in this case the strain due to moisture is anisotropic.

Because the deformations are small, we assume the material to be linearly elastic. Also we take the material to be homogeneous and isotropic with respect to elastic deformations. Therefore, we can use Hooke's law which relates the deformations to the stresses t_{ij}. This gives

$$e_{ij}^{(el)} = \frac{1+\nu}{E} t_{ij} - \frac{\nu}{E}\delta_{ij} t_{kk}, \tag{14}$$

where E is the elasticity modulus and ν is Poisson's ratio. Substituting Eqs. (**??**) and (**??**) into Eq. (**??**) yields the following set of equations for the total deformations in the plate

$$e_{xx} = \frac{1+\nu}{E} t_{xx} - \frac{\nu}{E}(t_{xx} + t_{zz}) + \alpha_x C, \tag{15a}$$

$$e_{xz} = \frac{1+\nu}{E} t_{xz}, \tag{15b}$$

$$e_{zz} = \frac{1+\nu}{E} t_{zz} - \frac{\nu}{E}(t_{xx} + t_{zz}) + \alpha_z C. \tag{15c}$$

Combining Eqs. (**??**) and (**??**) leads to the following set of equations which relate the stresses t_{ij} to the displacements u and w,

$$t_{xx} = \frac{E}{1-\nu^2}\left(\frac{\partial u}{\partial x} + \nu\frac{\partial w}{\partial z} - (\alpha_x + \nu\alpha_z)C\right), \tag{16a}$$

$$t_{xz} = \frac{E}{1+\nu}\frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right), \tag{16b}$$

$$t_{zz} = \frac{E}{1-\nu^2}\left(\frac{\partial w}{\partial z} + \nu\frac{\partial u}{\partial x} - (\alpha_z + \nu\alpha_x)C\right). \tag{16c}$$

Together with (**??**), this yields a coupled set of five first order linear partial differential equations. To complete this set of partial differential equations, we still need to derive boundary conditions. The boundary conditions are given by the following

$$t_{zz} = t_{xz} = 0, \qquad \text{on} \quad z = \pm h, \tag{17a}$$

$$t_{xx} = t_{xz} = 0, \qquad \text{on} \quad x = L, \tag{17b}$$

$$u = \frac{\partial w}{\partial x} = 0, \qquad \text{on} \quad x = 0. \tag{17c}$$

Here, Eqs. (??) and (??) are stress free boundary conditions and Eq. (??) represents the clamped end.

Since the aspect ratio is small, we try to find a solution by assuming that the normal stress component in the z-direction, t_{zz}, is negligible. With this assumption, it follows from (??), (??) and (??) that also t_{xx} and t_{xz} vanish everywhere. Using this, (??) gives us

$$\frac{\partial u}{\partial x} = \alpha_x C, \tag{18a}$$

$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0, \tag{18b}$$

$$\frac{\partial w}{\partial z} = \alpha_z C. \tag{18c}$$

Using $C = C(z)$, we obtain from Eqs. (??) and (??)

$$u = \alpha_x C(z) x + g(z), \tag{19a}$$

$$w = \alpha_z \int^{z} C(\xi) d\xi + l(x). \tag{19b}$$

The boundary condition on u in Eq.(??) gives us $g(z) \equiv 0$. Substituting Eqs. (??) and (??) into Eq. (??) gives us

$$\alpha_x C'(z) x + l'(x) = 0, \tag{20}$$

everywhere. This tells us that C has to be linear, i.e. $C(z) = A_1 + A_2 z$. This corresponds to the moisture profile after the panel has been exposed to a different concentrations on either face for long time periods, see Section 3. Using this, we can solve Eq. (??) for h, yielding

$$l(x) = -\frac{1}{2}\alpha_x A_2 x^2 + D, \tag{21}$$

where D is a constant representing translation. Taking $D = 0$ we end up with the following expressions for the displacements

$$u = \alpha_x (A_1 + A_2 z) x, \tag{22a}$$

$$w = \alpha_z \left(A_1 z + \frac{1}{2} A_2 z^2 \right) - \frac{1}{2}\alpha_x A_2 x^2. \tag{22b}$$

Note that the second boundary condition in Eq. (??) is satisfied. We note that the equations in this section are linear and w is only specified in terms of its derivatives, therefore the solution (??)-(??) is unique up to a translation in w. The centre line $z = 0$ is approximately a circular arc with radius given by $1/\alpha_x A_2$.

For a plate with a thickness of 1cm, and a length of 2m, the results are shown in Figure ??. We have taken the following $\alpha_x C(z) = 0.002 * (h + z)\ \mathrm{m}^{-1}$ and $\alpha_z C(z) = 0.03 * (h + z)\ \mathrm{m}^{-1}$ where h is half the thickness of the plate.

The solution (??)-(??) only allows us to deal with the concentration of water as a linear function of z. One possible technique for dealing with a general form for the concentration of water is asymptotics. An analysis was undertaken with the small parameter being the square of the aspect ratio. There is a boundary layer at the edge of the beam ($x = L$). The displacements were not determined at leading order in the outer expansion. Further research is required.

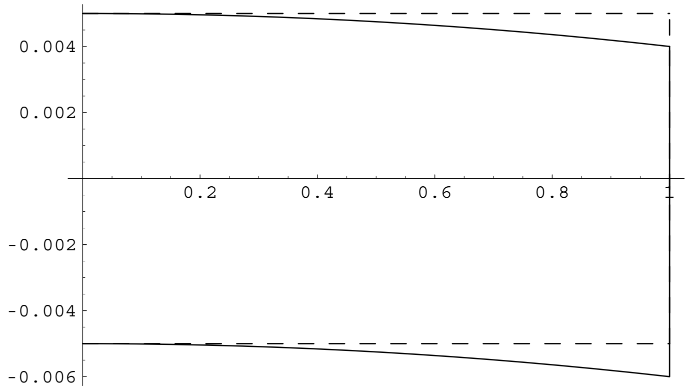

Figure 6: The warpage of a panel due to an asymmetric moisture content.

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