Modelling a long production line as a train with coupled carriages

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Abstract

In this paper, we model an active tension control (ATC) of a long production line offered by Marel Stork Poultry Processing. The production line consists of trolleys with poultry product which move along a rail. The trolleys are connected by a chain. The motor drives the chain by pushing the trolleys forward. However, when the motor is spinning too quickly the tension in the chain, right after the motor can drop which leads to collisions of the trolleys or entanglement of the chain. Hence, control of the tension is necessary. This ATC method works by means of a dead weight, pulling on the chain after the motor, the so-called 'dancer'. The dancer maintains the tension in the chain when the weight is sufficiently high. However, if the dancer is too heavy it will shorten the life-time of the chain. By modelling the motion of the trolleys between the first motor and the following dancer we numerically compute the optimal force for the dancer. Furthermore, we suggest extensions of our model and a novel modification to control the tension.

1 Introduction

In this paper we consider a problem proposed by Marel Stork Poultry Processing at SWI 2016. Marel Stork offers solutions for in-line poultry processing in accordance with a desired automation level and production capacities. Marel Stork systems are modular, that is, they can be combined with other equipment and with manual processes. We consider only one modular part of the whole processing system, namely a long production line transporting poultry through a chilled storage room. This closed-loop line is called an overhead conveyor.

The overhead conveyor consists of a long chain, the maximum length of which is 5500 m, driven by the up to 40 electric motors along the rail with a constant speed (0.6-0.8 m/s), see Figure 1.

Every 6 inches, a trolley is attached to the chain. These trolleys move over an overhead rail. Each trolley contains a shackle, hanging downwards, which suspends poultry products, see Figure 1. Each poultry product weighs between 1.5 and 2.5 kg. We will refer to a trolley containing poultry product as a carriage. The weight of a fully loaded production line is 68000 kg. The hanging poultry in the shackle can swing in all directions; also, the products can almost touch each other with their wings.
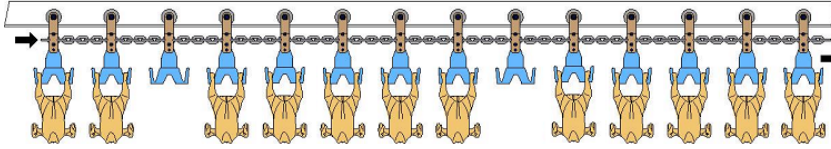
Figure 1: Schematics of the overhead conveyor with poultry. Courtesy of Marel Stork Poultry Processing

From the characteristics above one can see that the production line is very heavy and has a relatively large travelling speed. A problem occurring in the conveyor is slack of the chain. Slack has many causes, however, it is most often caused by the motor. The motor contains uniformly placed slots through which it transports the carriages, see Figure 2. If the motor is spinning quickly and if the chain is stretched due to wear such that the length of the chain between the carriages is longer than the distance between the slots then the chain will slack when carriages leave the motor. This slack could spread throughout the chain. If the chain slacks too much
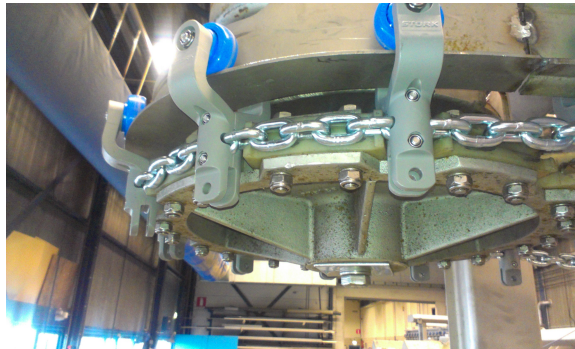

Figure 2: A motor of the overhead conveyor. Courtesy of Marel Stork Poultry Processing

the carriages can collide or the chain could entangle. This leads to a production stop and could even cause permanent damage of the production line. Hence, whenever too much slack is observed the production line is stopped.

The slack is controlled by an active tension control whose simplified schematic is displayed in Figure 3. Since the chain is very long many motors are needed to drive the chain. The first motor, the master motor, turns at a fixed speed. Each motor is followed by a dead weight, the so-called 'dancer'. The dancer is a pulley with a heavy weight. The pulley is fixed to a rail whose length determines the range of the motion of the dancer. Since the dancer applies a force it pulls the carriages in its direction and, consequently, reduces the slack. Hence, it is very similar to a train locomotive
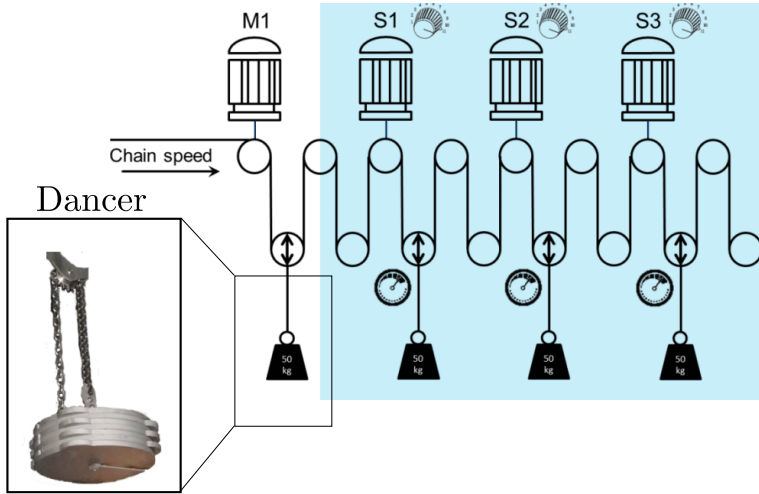
Figure 3: Active tension control of the overhead conveyor. M1, S1, S2 and S3 denote motors. The motor M1 is the master motor which turns at a fixed speed. Each motor is followed by a dead weight, the 'dancer'. The dancer is a pulley with a heavy weight. Since the dancer applies a force it pulls the carriages in its direction and, consequently, reduces the slack. Observe that a quantitative measure of the slack in the chain between a motor and the following dancer is given by the position of the dancer. The motors S1, S2 and S3 are the slave motors. Their speed is controlled by the position of the dancer. Courtesy of Marel Stork Poultry Processing

which pulls its carriages forward. However, contrary to the carriages of a locomotive, these carriages will move past the pulling force. Observe that a quantitative measure of the slack in the chain between a motor and the following dancer is given by the position of the dancer. Hence, the speed of the motors after the master motor are controlled by the position of the dancer. Thus, the motors after the master motor are called slave motors.

Observe that the dancer does not make the slack vanish instantaneously. Hence, the force of the dancer should be large enough such that it counteracts the slack caused by the motor. However, the problem with the dancer is that it applies a constant force on the chain which leads to stretching of the chain over time, in turn, this leads to more slack. Furthermore, when the chain is stretched for about 3%, the chain is classified as worn out and needs to be entirely replaced. During this maintenance process no products can be processed. Thus, it is of importance to maintain a tension in the chain which prevents its slack and reduces its wear.

In Section 2 we set up a model for the motion of the carriages where the slack is caused by the motor. More specifically, we focus on the motion between the master

motor and the following dancer. In Section 3 we present the results of numerical
simulations of the model constructed in Section 2. Finally, we present the conclusions
and recommendations in Section 4. More specifically, we explain how the numerics
validates our model, how the model can be extended to model the system better and
how the mathematical results might lead to an improvement of the current system.

2   Modelling the production line as a train with carriages

In this section we formulate the model. We consider a chain that is fully loaded with
product. However, we only study part of the complete chain and restrict our model
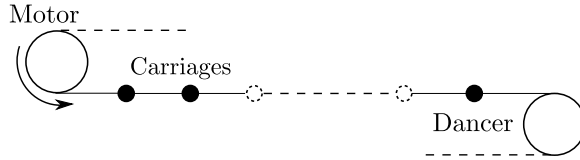to the study of the carriage motion between the master motor and the next dancer,
see Figure 4.

Figure 4: The chain between the master motor and the dancer. The carriages are
depicted by the black dots.

2.1   Modelling assumptions

To set up the equations of motion we consider some modelling assumptions:

Carriage: 1. Carriages are point masses with equal mass.

2. The friction of the wheels with the rails is linear.

3. When the chain between two carriages has slack the carriages do not
exert forces on each other. However, when the chain is tight the
forces on both carriages are equal.

Dancer: 1. The position of the dancer is fixed.

2. The dancer applies a constant force on the chain.

Chain: 1. There is no mechanical energy loss through the chain.

Motor: 1. The motor supplies carriages with a constant rate.

2. The carriages that leave the motor have a prescribed initial velocity.
This velocity is sufficiently low such that the chain between the motor
and the first carriage is never pulled tight whenever this carriage is
unaffected by the dancer. Hence, the motor causes slack.

The model will consist of equations of motion for each of the carriages. Here we measure their position by the distance to the motor where the motor is located at $x = 0$ and and the dancer is located at $x = \ell$. Based on the assumptions above we can introduce the parameters for the equations of motion:

ℓ: the length, in meters, from the motor to the dancer

ℓ_{ct}: the length, in meters, of the chain between two carriages when it is pulled tight

m: the mass, in kilograms, of the (loaded) carriage

c: the friction coefficient, in kilograms per second, between the wheels of the hangar with the rails

f_{mot}: the number of carriages that enter per second through the motor

v_{mot}: the velocity, in meters per second, of the carriages that enter through the motor

F_{dan}: the force, in Newton, which is being applied by the dancer

We assume that all the constants are non-zero.

Denote by $x(t)$ the position of a carriage. Then, if the carriages is unaffected by the force of the dancer the equation of motion is given by

$$m\ddot{x} = -c\dot{x}, \tag{1}$$

where we used the short-hand notation $\dot{x} = dx/dt$ and $\ddot{x} = d^2x/dt^2$. If the carriage is part of W carriages whose chains are pulled tight the total mass is Wm and the total friction is Wc. Hence, if these carriages are affected by the force of the dancer the equation of motion is given by

$$Wm\ddot{x} = -Wc\dot{x} + F_{\mathrm{dan}}. \tag{2}$$

The equations (1) and (2) are standard ODEs of the form

$$\ddot{y} = a\dot{y} + b, \tag{3}$$

with $a, b \in \mathbb{R}$ of which solutions are given by

$$y(t) = \frac{c_1 \mathrm{e}^{at} - bt}{a} + c_2, \tag{4}$$

with $c_1, c_2 \in \mathbb{R}$ constants determined by the initial conditions. It will turn out that all the ODEs in this paper are of the form (3).

Observe that assumption 2 for the motor is an assumptions on the parameters. Hence, let us formulate this assumptions in terms of the parameters using (1):

Motor: 2. Consider the equations of motion in (1). Let x satisfy the initial conditions

$$x(0) = 0, \ \dot{x}(0) = v_{\mathrm{mot}}.$$

At $t = 1/f_{\mathrm{mot}}$ a new carriage will enter via the motor. Hence, we assume that v_{mot} is sufficiently small such that

$$x(1/f_{\mathrm{mot}}) < \ell_{ct}.$$

2.2 Set-up: possible events

We assumed that when the chain between two carriages is loose the carriages do not exert forces on each other. However, when the chain is tight the forces on both carriages are equal. In formulating the model it turns out that the configuration of the carriages is very important. As we explained before the chain between the carriages can hang loose or be tight and we have to take that into account. So, by configuration we mean the number of carriages between the motor and dancer, and also between which carriages the chain is tight and loose.

For a given configuration the equations of motion remain unchanged. However, when the configuration changes the equations of motion also change. The configuration changes when one of the following events occur:

P: a loose chain is pulled tight,

C: a collision occurs between two carriages,

E: a new carriage enters via the motor,

L: the last carriage leaves by passing the dancer.

We shall abbreviate these events by the capital letters above. Note that several of the above events could also occur at the same time. Hence, all possible events are given by all the combinations of (P, C, E, L). When an event occurs the equations of motion change and we have to prescribe the new equations of motion until the next event. The event C is special in this respect since in this case the production line should be stopped so our model should end too.

2.3 The event map

In this section we will present a general procedure which gives the equations of motion as the system undergoes a configuration change due to an event.

First, we assume that we know the equations of motion at $t = 0$ and that at $t = 0$ there are $N_0 > 0$ carriages between the motor and dancer. We denote the position of the ith carriage by $x_{0i}(t)$ for $i = 1, 2, \ldots, N_0$. Here we order the carriages in such a way that the 1st carriage is closest to the motor and the N_0th carriage is closest to the dancer, see Figure 5.
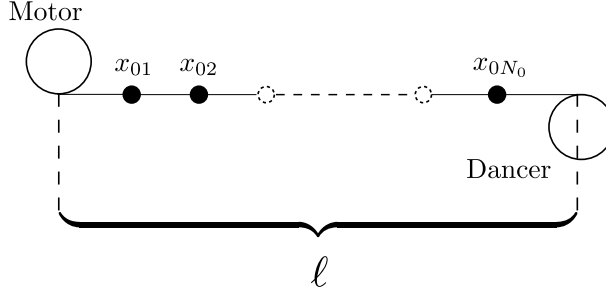
Figure 5: Carriage position at $t = 0$. The carriages are depicted by black dots. The 1st carriage is closest to the motor and the N_0th carriage is closest to the dancer.

Now, assume that the first time that an event occurs is at $t_1 > 0$. Then, a full description of the carriages for $0 \leq t < t_1$ is given by the tuple

$$(T_0, X_0), \tag{5}$$

with

$$T_0 := [0, t_1), \quad X_0(t) = (x_{01}, x_{02}, \cdots, x_{0N_0}).$$

The tuple (5) will be called the carriage motion on T_0. Denote the event at t_1 by A. Then we want to construct a map Ψ such that

$$\Psi(T_0, X_0) = (T_1, X_1),$$

with

$$T_1 := [t_1, t_2), \quad X_1(t) = (x_{11}, x_{12}, \cdots, x_{1N_1}),$$

where t_2 is time when the next event occurs, N_1 is the number of carriages between the motor and dancer and $x_{1i}(t)$ is the position for the ith carriage for all $t \in T_1$. Again the carriages are ordered in such a way that the 1st carriage is closest to the motor and the N_1th carriage is closest to the dancer. Observe that Ψ maps (T_0, X_0) into the carriage motion until the next event. Hence, we call Ψ the event map and $\Psi(T_0, X_0)$ is called the carriage motion from the 1st to the 2nd event of (T_0, X_0). Similarly, $\Psi^n(T_0, X_0)$ is called the carriage motion from the nth to the $(n + 1)$th event of (T_0, X_0).

2.4   Initial carriage motion

To construct a general Ψ is a very lengthy exercise. However, by restricting to a specific initial carriage motion the construction of Ψ simplifies. We consider (T_0, X_0) the tuple (5). As the initial configuration we assume that the chain is tight between all the carriages. Observe that we must require that $N_0 \ell_{ct} \leq \ell < (N_0 + 1)\ell_{ct}$ when all

the chains are tight. Then, it follows from (2) that the equations of motion for the carriages are given by

$$N_0 m \ddot{x}_{0j} = -N_0 c \dot{x}_{0j} + F_{\text{dan}}, \; j = 1, 2, \cdots, N_0 \;, \tag{6}$$

with initial conditions

$$x_{0j}(0) = (j-1)\ell_{ct} + d_0, \qquad \dot{x}_{0j}(0) = v_1 > 0, \; j = 1, 2, \cdots, N_0 \;, \tag{7}$$

where $d_0 \in [0, \ell_{ct}]$. Since the chain between the carriages is tight, the initial distance between two adjacent carriages is ℓ_{ct}. Observe that d_0 is the distance between the motor and the first carriage. Hence, the chain between the motor and the first carriage can be loose ($d_0 < \ell_{ct}$), tight ($d_0 = \ell_{ct}$) or the first carriage starts at the position of the motor ($d_0 = 0$). Since the N_0 carriages move as a whole all the carriage have the same initial velocity.

Next, we look at which event can occur at t_1. First, let us make a distinction between two different P events:

P_c: The chain between two carriages is pulled tight

P_{mot}: The chain between the motor and the first carriage is pulled tight

Then, the events C and P_c cannot occur at t_1. Hence, the events E, L and/or P_{mot} can occur at t_1. We assume that the motor is switched on at $t = 0$. Hence, at $t = 1/f_{\text{mot}}$ the first carriage will enter via the motor. Thus, t_1 is given by

$$t_1 = \min(\tau_1, \tau_2, 1/f_{\text{mot}}), \tag{8}$$

where $\tau_1, \tau_2 > 0$ are the smallest τ_1, τ_2 such that

$$x_{0N_0}(\tau_1) = \ell, \; (\text{event } L)$$
$$x_{01}(\tau_2) = \ell_{ct} \; (\text{event } E).$$

2.5 Carriage motion from the nth to the $(n+1)$th event

It turns out that we chose the initial carriage motion, (T_0, X_0), in such a way that the following holds for $\Psi^n(T_0, X_0) = (T_n, X_n)$ for any $n \in \mathbb{N}$:

Property 1. The carriages can be divided into two connected parts: the loose part, which consists of all the carriages with a loose chain in between them, and the tight part, which consists of all the carriages with a tight chain between them. Denote the number of carriages of the loose part by K_n. If the loose part contains more than 0 carriages, so $K_n > 0$, then the first carriage of the loose part connects to the motor. If the tight part consists of more than 0 carriages then the last carriage of the tight part is next to the dancer, see Figure 6.

Property 2. If $K_n > 0$ then the speed of the carriage closest to the motor is largest and the speed of the carriages decreases when moving away from the motor. Hence, for all $t \in T_n$ the following inequality holds:

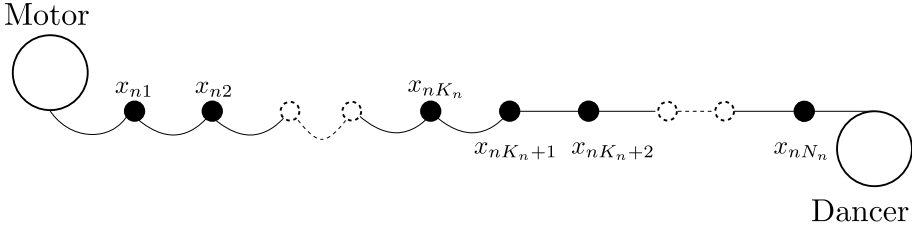$$\frac{dx_{n1}}{dt}(t) > \frac{dx_{n2}}{dt}(t) > \cdots > \frac{dx_{nK_n}}{dt}(t). \tag{9}$$

Figure 6: The loose part and tight part. Denote the number of carriages by N_n and the carriages of the loose part by K_n. The carriages can be divided into a loose part with K_n carriages which are connected to the motor and a tight part of $(N_n - K_n)$ carriages which are connected to the dancer.

These two properties are important for the construction of Ψ. We will now construct Ψ and prove the above properties by induction. Observe that property 1 and 2 hold for (T_0, X_0).

Now, we assume that (T_k, X_k) is known. Furthermore, we assume that (T_k, X_k) satisfies property 1 and 2. Then, we want to prescribe the carriage motion from the $(k+1)$th to the $(k+2)$th event of (T_0, X_0), $\Psi(T_k, X_k) =: (T_{k+1}, X_{k+1})$, and prove that property 1 and 2 hold. We denote $T_k = [t_k, t_{k+1})$ and N_k is the number of carriages. As before, we index X_k in the following way:

$$X_k = (x_{k1}, x_{k2}, \cdots, x_{kN_k}).$$

First we determine which events can occur at t_{k+1}. We will present mathematical equivalents for the events. Using property 1 and 2, we first make some observations:

- If P_{mot} occurs at t_{k+1} then all the chains are pulled tight.

- If P_c occurs at t_{k+1} then the chain between the loose part and the tight part of the carriages is pulled tight.

- If C occurs at t_{k+1} then the last carriage of the loose part and the first carriage of the tight part collide.

Denote the number of carriages of the loose part by K_k. Then the events are described by

$$P_{\text{mot}} \text{ a } t_{k+1} \iff x_{k1}(t_{k+1}) = \ell_{ct}, \tag{10}$$

$$L \text{ at } t_{k+1} \iff x_{kN_k}(t_{k+1}) = \ell, \tag{11}$$

$$E \text{ at } t_{k+1} \iff t_{k+1} = p/f_{\text{mot}} \text{ with } p = \min_{\substack{p_0/f_{\text{mot}} - t_k > 0, \\ p_0 \in \mathbb{N}}} p_0, \tag{12}$$

and if $K_k > 0$ then, in addition, the following events could occur:

$$P_c \text{ at } t_{k+1} \iff x_{kK_k+1}(t_{k+1}) - x_{kK_k}(t_{k+1}) = \ell, \tag{13}$$

$$C \text{ at } t_{k+1} \iff x_{kK_k+1}(t_{k+1}) - x_{kK_k}(t_{k+1}) = 0. \tag{14}$$

If C occurs at t_{k+1} then there is no carriage motion after the $(k+1)$th event of (T_0, X_0). At t_{k+1} either a single event or a combination of $(L, E, P_{\text{mot}}, P_c)$ can occur. Next, we will prescribe X_{k+1} based on which event occurs:

Event L:

The last carriage leaves. We have that

$$X_{k+1} = (x_{k1}, x_{k2}, \cdots x_{kK_k}, y_{K_k+1}, y_{K_k+2}, \cdots, y_{N_k-1}), \tag{15}$$

where y_j with $j = K_k + 1, \cdots, N_k - 1$, satisfies

$$m(N_k - 1 - K_k)\ddot{y}_j = -c(N_k - 1 - K_k)\dot{y}_j + F_{\text{dan}}, \tag{16}$$

and initial conditions

$$y_j(t_{k+1}) = x_{kj}(t_{k+1}), \quad \dot{y}_j(t_{k+1}) = \dot{x}_{kj}(t_{k+1}). \tag{17}$$

Event E:

A new carriage enters. Hence, we have

$$X_{k+1} = (z, x_{k1}, x_{k2}, \cdots, x_{kN_k}), \tag{18}$$

with z satisfying

$$m\ddot{z} = -c\dot{z}, \tag{19}$$

and initial conditions

$$z(t_{k+1}) = 0, \quad \dot{z}(t_{k+1}) = v_{\text{mot}}. \tag{20}$$

Events E and L:

We have that

$$X_{k+1} = (z, x_{k1}, x_{k2}, \cdots x_{kK_k}, y_{K_k+1}, y_{K_k+2}, \cdots, y_{N_k-1}), \tag{21}$$

where z satisfies Equation (19) and (20) and where y_j with $j = K_k + 1, \cdots, N_k - 1$, satisfies (16) and (17).

Event P_{mot}:

We have that
$$X_{k+1} = (w_1, w_2, \cdots, w_{N_k}),$$

where w_j with $j = 1, 2, \cdots, N_k$, satisfies

$$\dot{w}_j = 0, \tag{22}$$

and initial conditions
$$w_j(t_{k+1}) = x_{kj}(t_{k+1}). \tag{23}$$

Events E and P_{mot}:

We have that
$$X_{k+1} = (w_0, w_1, w_2, \cdots, w_{N_k}),$$

where w_j with $j = 0, 1, 2, \cdots, N_k$, satisfies Equation (22) and (23).

Events L and P_{mot}:

We have that
$$X_{k+1} = (w_1, w_2, \cdots, w_{N_k-1}),$$

where w_j with $j = 1, 2, \cdots, N_k - 1$ satisfies (22) with (23).

Events E, L and P_{mot}:

We have that
$$X_{k+1} = (w_0, w_1, w_2, \cdots, w_{N_k-1}),$$

where w_j with $j = 0, 1, 2, \cdots, N_k - 1$, satisfies (22) and (23).

Event P_c:

This only happens when $K_k > 0$. The K_kth carriage which was in the loose part during T_k will become part of the tight part at t_{k+1}. We then have that

$$X_{k+1} = (x_{k1}, x_{k2}, \cdots, x_{kK_k-1}, u_{K_k}, u_{K_k+1}, \cdots, u_{N_k}), \tag{24}$$

where u_j with $j = K_k, K_k + 1, \cdots, N_k$, satisfies

$$m(N_k - K_k + 1)\ddot{u}_j = -c(N_k - K_k + 1)\dot{u}_j + F_{\text{dan}}, \tag{25}$$

and initial conditions

$$u_j(t_{k+1}) = x_{kj}(t_{k+1}), \quad \dot{u}_j(t_{k+1}) = \dot{x}_{kj}(t_{k+1}). \tag{26}$$

Events E and P_c:

Again, $K_k > 0$ and it follows that

$$X_{k+1} = (z, x_{k1}, x_{k2}, \cdots, x_{kK_k-1}, u_{K_k}, u_{K_k+1}, \cdots, u_{kN_k}), \tag{27}$$

where z is satisfies (19) and (20) and where u_j with $j = K_k, K_k + 1, \cdots, N_k$ satisfies (25) and (26).

Events L and P_c:

Then $K_k > 0$ and

$$X_{k+1} = (x_{k1}, x_{k2}, \cdots, x_{kK_k-1}, \hat{u}_{K_k}, \hat{u}_{K_k+1}, \cdots, \hat{u}_{N_k-1}), \tag{28}$$

where $\hat{u}_j$ with $j = K_k, K_k + 1, \cdots, N_k - 1$ satisfies

$$m(N_k - 1 - K_k)\ddot{\hat{u}}_j = -c(N_k - 1 - K_k)\dot{\hat{u}}_j + F_{\text{dan}} \tag{29}$$

and initial conditions

$$\hat{u}_j(t_{k+1}) = x_{kj}(t_{k+1}), \quad \dot{\hat{u}}_j(t_{k+1}) = \dot{x}_{kj}(t_{k+1}). \tag{30}$$

Events E, L and P_c:

Then $K_k > 0$ and

$$X_{k+1} = (z, x_{k1}, x_{k2}, \cdots, x_{kK_k-1}, \hat{u}_{K_k}, \hat{u}_{K_k+1}, \cdots, \hat{u}_{N_k-1}), \tag{31}$$

where z is satisfies (19) with (20) and where $\hat{u}_j$ with $j = K_k, K_k + 1, \cdots, N_k - 1$ satisfies (29) with (30). The ODEs above are solved by (4).

We have given X_{k+1}. We denote $T_{k+1} = [t_{k+1}, t_{k+2})$ and N_{k+1} is the number of carriages. As before, we index X_{k+1} in the following way:

$$X_{k+1} = (x_{k+1\ 1}, x_{k+1\ 2}, \cdots, x_{k+1\ N_{k+1}}).$$

It remains to give t_{k+2}. From (10), (11), (12), (13), (14) it follows that

$$t_{k+2} = \begin{cases} \min(\tau_1, \tau_2, \tau_3) & \text{if } K_{k+1} = 0, \\ \min(\tau_1, \tau_2, \tau_3, \tau_4\tau_5) & \text{if } K_{k+1} > 0, \end{cases}$$

where $\tau_i > t_{k+1}$ with $i = 1, 2, 3, 4, 5$ are the smallest τ_i such that

$$
\begin{aligned}
x_{k+1\ 1}(\tau_1) &= \ell_{ct}, \\
x_{k+1\ N_{k+1}}(\tau_2) &= \ell, \\
\tau_3 \bmod 1/f_{\text{mot}} &= 0, \\
x_{k+1\ K_k+1}(\tau_4) - x_{k+1\ K_k}(\tau_4) &= \ell, \\
x_{k+1\ K_k+1}(\tau_5) - x_{k+1\ K_k}(\tau_5) &= 0.
\end{aligned}
$$

We have given T_{k+1}. We are left with proving property 1 and 2, but this follows directly from considering the equations of motion for all the cases.

3   Numerical simulation

We implemented the model in Mathematica. It turns out that the dynamics of the slack caused by the motor does not change if we consider a chain with many carriages. Hence, for convenience we consider a chain starting with only 4 carriages. At $t = 0$ we assume that the system is at rest, which means that the velocity of the carriages is zero. In the physical system many carriages per minute enter via the motor. Hence, we will take f_{mot} large. We consider the following parameters:

$$\ell = 4, \ \ell_{ct} = 1, \ c = 1/100, \ m = 1, \ f_{\mathrm{mot}} = 100, \ v_{\mathrm{mot}} = 10, \tag{32}$$

and for the following initial conditions:

$$X_0(0) = (1/2, 3/2, 5/2, 7/2), \ \dot{X}_0(0) = (0, 0, 0, 0). \tag{33}$$

We will vary the force F_{dan}. The results for $F_{\mathrm{dan}} = 150$ and $F_{\mathrm{dan}} = 250$ are displayed in Figure 7 and Figure 8, respectively. In Figure 7 and Figure 8 the carriages are labelled with numbers so that the motion of each carriage can be followed over time.
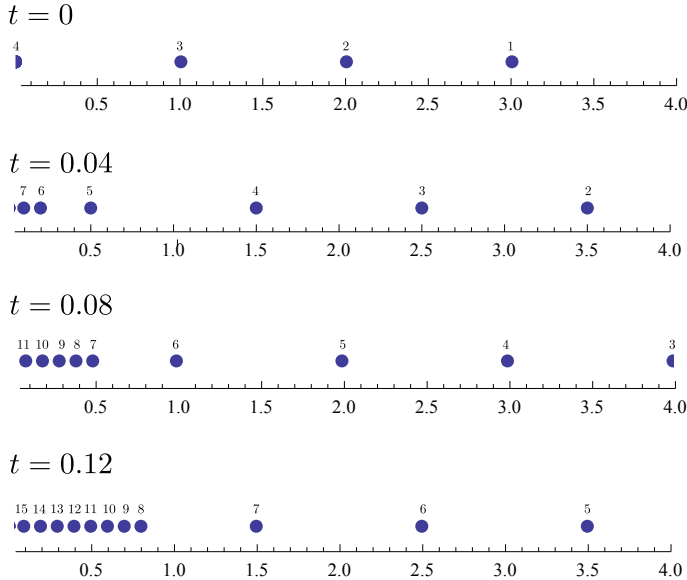

Figure 7: Time frames of the numerical simulations with initial conditions (33) and with parameters (32), $F_{\mathrm{dan}} = 150$.
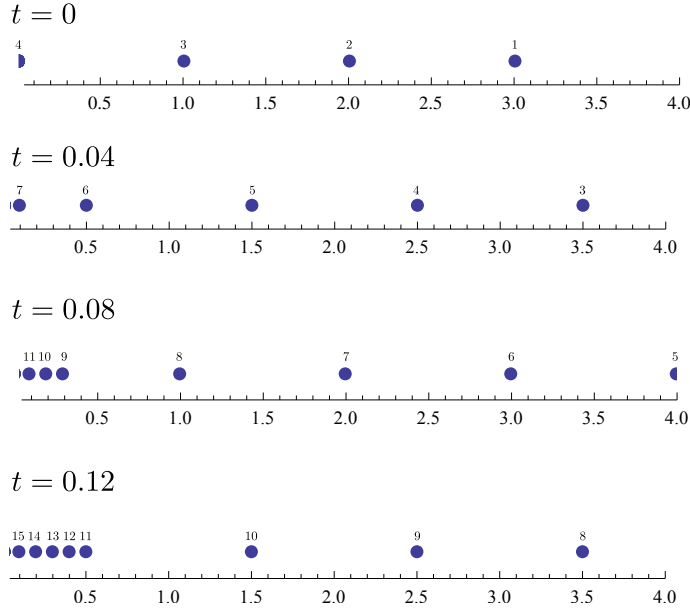
Figure 8: Time frames of the numerical simulations with initial conditions (33) and with parameters (32), $F_{\text{dan}} = 250$.

In Figure 7 and Figure 8 the carriages with loose chain between them are pulled tight by the dancer. We observe that the time it takes for the chain between the loose part and the tight part of the carriages to be pulled tight is too large. This can be seen from the fact that the loose carriages accumulate over time. We find that for $F_{\text{dan}} = 390$ the carriages have the same configuration as for $F_{\text{dan}} = 250$ at $t = 0.12$ at a later time, namely at $t = 2$, see Figure 9.
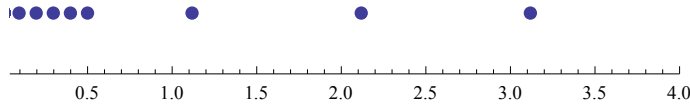

Figure 9: The numerical simulation at $t = 2$ with initial conditions (33), parameters (32) and $F_{\text{dan}} = 390$.

If we take $F_{\text{dan}} \geq 400$ then we find that the events P_{mot} and L are the first events that occur. Recall that when P_{mot} occurs all the velocities become zero. The event $P_{\text{mot}}L$ is followed by the event E after which we are back in the starting configuration and the process repeats. Consequently, there is no accumulation of carriages with loose chain.

4   Conclusions and recommendations

In this paper we constructed a mathematical model for the carriage motion between the master motor and the next dancer. In Section 3 we found that if the force applied by the dancer is large enough then the slack in the chain that is caused by the motor will not spread over the whole chain. This is also observed in the physical system. Alternatively, we could have modelled the chain as a moving continuum (e.g., string Chen (2005)) or a harmonic oscillator (spring-mass system). However, we looked into the approaches and neither of them yielded satisfying results.

The model we formulated is only a first step in the study of this system. There are several ways in which it can be extended. As next steps, we recommend the following extensions of our model:

- **General initial configurations:** Our model can only be used for the initial configuration when the chain is tight between all the carriages. For a more general initial configuration the possible events will increase. Consequently, the event map Ψ will become more complicated.

- **Moving dancer**: The distance between the motor and dancer is assumed to be constant in our model. However, in the physical system the dancer can move and this can be incorporated in our model.

- **Control problem for coupled dancers and motors:** In the original problem the master motor is followed by motors whose speed is coupled to the position of the dancer. Using our model we can formulate a control problem for this system.

- **Not fully loaded chain**: When there is not a product on every trolley but several are empty we should consider a partially filled chain. This can be done by varying the mass of the carriages which enter via the motor.

- **Realistic parameters**: In Section 3 we only consider very specific parameters. Parameters which better fit the reality should be studied.

The numerical simulation in Section 3 is aimed at finding the lowest force on the dancer such that the carriages with loose chains between them do not accumulate. We call this the optimal force of the dancer. Recall that the greater the force of the dancer the shorter the life-time of the chain. A topic for future work is a further improvement of the life-time of the chain. More specifically, by modifying the system we want to take the force of the dancer lower than the optimal force of our model while ensuring that the entire chain will not slack over time. If we could reduce the speed of the carriages with loose chain that leave the motor then the dancer has more time to pull the chains between the carriages tight. This might be accomplished by placing a high friction mat on the rails close to the motor. This reduces the speed of the carriages close to the motor. By using our model it is possible to test whether this might work.

References

L. Chen. Analysis and control of transverse vibration of axially moving strings. *Applied Mechanics Reviews*, 2005.

Courtesy of Marel Stork Poultry Processing.